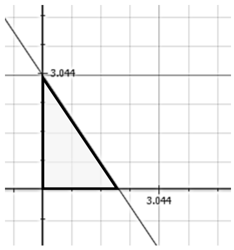
ANSWERS WITH SOLUTIONS

1. Find the value of $\sqrt{x+\sqrt{14-x}}$ when $x = -2$.

$$\sqrt{-2+\sqrt{14-(-2)}}$$
$$= \sqrt{-2+\sqrt{14+2}}$$
$$= \sqrt{2}$$

ANSWER: $\sqrt{2}$

2. Find the area of the triangle formed by the coordinate axes and the line $3x + 2y = 6$.



The triangle has a base equal to the x-intercept of the graph of $3x + 2y = 6$ and a height equal to the y-intercept. The area, therefore is half the product of the intercepts.

$$A = \frac{1}{2}bh \quad A = \frac{1}{2}(2)(3) \quad A = 3 \; square\ units$$

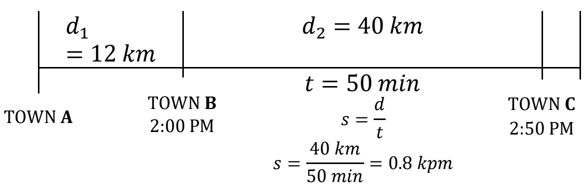
ANSWER: 3 square units

3. A sequence is defined by $a_n = 3(a_{n-1} + 2)$ for $n \geq 2$, where $a_1 = 1$. What is a_4?

$$a_1 = 1$$
$$a_2 = 3(a_1 + 2) \quad a_2 = 3(1+2) \quad a_2 = 9$$
$$a_3 = 3(a_2 + 2) \quad a_3 = 3(9+2) \quad a_2 = 33$$
$$a_4 = 3(a_3 + 2) \quad a_4 = 3(33+2) \quad a_2 = 105$$

ANSWER: 105

4. Christa leaves Town A. After traveling 12 km, she reaches Town B at 2:00 P.M. Then she drove at a constant speed and passes Town C, 40 km from Town B, at 2:50 P.M. Find the function $d(t)$ that models the distance (in km) she has traveled from Town A t minutes after 2:00 P.M.



$$d = st \quad d = 0.8t \; for\ distance\ after\ Town\ B$$
$$d(t) = 12 + 0.8t$$

ANSWER: $\boldsymbol{d(t) = 12 + 0.8t}$

5. What is the largest negative integer that satisfies the inequality $|3x + 2| > 4$?

Two conditions will satisfy this inequality,
(1) If $3x + 2 > 4$; and
(2) If $3x + 2 < -4$
In case (1), $x > \frac{2}{3}$. In case (2), $x < -2$. This is the solution set to the inequality. Considering case (2), the largest negative integer that satisfies this condition is -3.
ANSWER: -3

6. A person has two parents, four grandparents, eight grand-parents, and so on. How many ancestors does a person have 10 generations back?

This being a geometric series of 2+4+8+...+a_{10},
Use the formula, $S_n = a_1\left(\frac{1-r^n}{1-r}\right)$

$$S_{10} = 2\left(\frac{1-2^{10}}{1-2}\right)$$
$$S_{10} = 2046$$

ANSWER: 2046

7. In ΔABC, $\angle B$ is twice $\angle A$, and $\angle C$ is three times as large as $\angle B$. Find $\angle C$
If $m\angle B = 2\,m\angle A \quad and \quad m\angle C = 3\,m\angle B$, then

$$m\angle C = 6\,m\angle A$$
$$m\angle A + m\angle B + m\angle C = 180°$$
$$m\angle A + 2\,m\angle A + 6\,m\angle A = 180°$$
$$9\,m\angle A = 180°$$
$$m\angle A = 20°$$
$$m\angle C = 6\,m\angle A = 120°$$

ANSWER: 120°

8. If $-3 \leq x \leq 0$, find the minimum value of $f(x) = x^2 + 4x$.

The graph of $f(x)$ opens upward based on its leading coefficient. Therefore, the absolute minimum value of the function is the y-coordinate, ***k***, of its vertex.

$$k = c - \frac{b^2}{4a}$$
$$k = 0 - \frac{4^2}{4(1)}$$
$$k = -4$$

ANSWER: -4

9. The 9th and the 11th term of an arithmetic sequence are 28 and 45, respectively. What is the 12th term?

$$2d = a_{11} - a_9 \qquad d = \frac{a_{11} - a_9}{2}$$
$$d = \frac{45 - 28}{2} = \frac{17}{2} \text{ or } 8.5$$
$$a_{12} = a_{11} + d \qquad a_{12} = 45 + 8.5$$

ANSWER: 53.5

10. Perform the indicated operation, and simplify:

$$\frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2 - x}$$
$$\frac{2(x-1) + 3x - 4}{x^2 - x}$$
$$\frac{5x - 6}{x^2 - x}$$

ANSWER: $\boldsymbol{\frac{5x-6}{x^2-x}}$

11. Find the range of the function $f(x) = |2x + 1|$.
For any absolute value function of this form, $f(x) = |g(x)|$ where $g(x)$ is a linear function, the range is always $\{y | y \in \mathbb{R}, y \geq 0\}$.
ANSWER: $\boldsymbol{y \geq 0}$

12. Find the value of $[x - (x - x^{-1})^{-1}]^{-1}$ when $x = 2$.

$$[2 - (2 - 2^{-1})^{-1}]^{-1}$$
$$\left[2 - \left(2 - \frac{1}{2}\right)^{-1}\right]^{-1}$$
$$\left[2 - \left(\frac{3}{2}\right)^{-1}\right]^{-1} = \left[2 - \frac{2}{3}\right]^{-1} \rightarrow \left(\frac{4}{3}\right)^{-1} = \frac{3}{4}$$

ANSWER: $\boldsymbol{\frac{3}{4}}$

13. Find the equation (in the form $ax + by = c$) of the line through the point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.

$$4x + 6y + 5 = 0 \rightarrow y = -\frac{2}{3}x - \frac{5}{6}$$

Solve for b given that $m = -\frac{2}{3}$ and $(5, 2)$

$$y_1 = mx_1 + b$$
$$2 = -\frac{2}{3}(5) + b \qquad b = \frac{16}{3}$$
$$y = -\frac{2}{3}x + \frac{16}{3} \rightarrow 2x + 3y = 16$$

ANSWER: $\boldsymbol{2x + 3y = 16}$

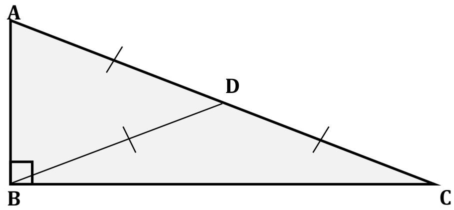
Note on an alternative:
When asked for the equation of a line, $ax + by = c$, **parallel** to a given linear equation of the form $\boldsymbol{a_1}x + \boldsymbol{b_1}y = c_1$ and passing through the point, $(\boldsymbol{x_1}, \boldsymbol{y_1})$, use the algorithm:

$$\boldsymbol{a_1 x + b_1 y = a_1 x_1 + b_1 y_1}$$

When asked for the equation of a line, $ax + by = c$, **perpendicular** to a given linear equation of the form $\boldsymbol{a_1}x + \boldsymbol{b_1}y = c_1$ and passing through the point, $(\boldsymbol{x_1}, \boldsymbol{y_1})$, use the algorithm:

$$\boldsymbol{b_1 x - a_1 y = -b_1 x_1 + a_1 y_1}$$

14. In ΔABC, $\angle B = 90°$, $\angle ACB = 28°$, and D is the midpoint of AC. What is $\angle BDC$?



The median to the hypotenuse of a right triangle is half the measure of the hypotenuse.
Since $BD = DC$, ΔBDC is isosceles. It also follows that $m\angle C = m\angle DBC$.

$$m\angle C + m\angle DBC + m\angle BDC = 180°$$
$$28° + 28° + m\angle BDC = 180°$$
$$m\angle BDC = 180° - 56°$$
$$m\angle BDC = 124°$$

ANSWER: 124°

15. Find all solutions of the system:

$$\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases}$$

$$2x + y = 1 \rightarrow y = -2x + 1$$

By substitution, $3x + 4(-2x + 1) = 14$

$$3x - 8x + 4 = 14$$
$$-5x = 10, \quad x = -2$$
$$y = -2x + 1 \rightarrow y = -2(-2) + 1$$
$$y = 5$$

ANSWER: $\boldsymbol{x = -2, y = 5}$

16. If $f(2x - 1) = x$, what is $f(2)$?

$$f(2x - 1) = f(2)$$
$$2x - 1 = 2 \qquad x = \frac{3}{2}$$

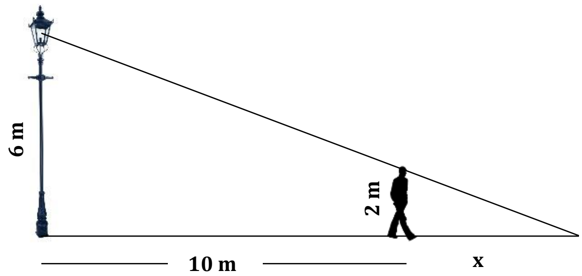
ANSWER: $\boldsymbol{x = \frac{3}{2}}$

17. What is the quotient when $3x^5+5x^4-4x^3+7x+3$ is divided by $x+2$?

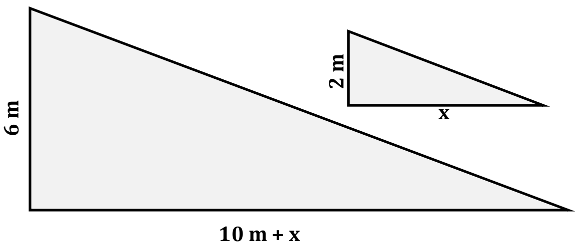
-2	3	5	-4	0	7	3
		-6	2	4	-8	2
	3	**-1**	**-2**	**4**	**-1**	5

ANSWER: $\boldsymbol{3x^4-x^3-2x^2+4x-1}$

18. A man is walking away from a lamppost with a light source 6 meters above the ground. The man is 2 meters tall. How long is his shadow when he is 10 meters from the lamppost?



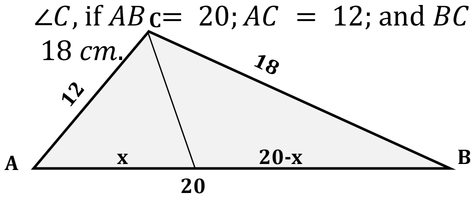
The two triangles are similar, based on the Triangle Proportionality Theorem.



$$\frac{6}{2}=\frac{10+x}{x}$$
$$6x=20+2x\ \rightarrow x=5$$

ANSWER: 5 $\boldsymbol{m}$

19. Find the length of the shorter segment made on side AB of ΔABC by the bisector of $\angle C$, if $AB=20$; $AC=12$; and $BC=18\ cm$.



By the Angle Bisector Theorem,
$$\frac{12}{18}=\frac{x}{20-x}\ \rightarrow\ \frac{2}{3}=\frac{x}{20-x}$$
$$40-2x=3x\ \rightarrow\ x=8$$

ANSWER: 8 $\boldsymbol{cm}$

20. How many different three-digit numbers less than 300 can be formed with the digits 1,2,3 and 5 if repetition of digits is not allowed?

2 possible digits: 1 or 2	**3** possible digits: One from (1 or 2), 3, or 5	**2** possible digits: Whichever digits are left

By the fundamental principle of counting, the product of 2, 3, and 2 is the answer.
ANSWER: 12

21. Factor completely the expression x^3-7x+6.
Use the rational zeroes theorem. Employ trial and error.

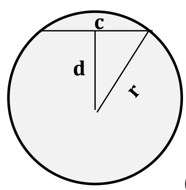
1	1	0	-7	6
		1	1	-6
	1	1	-6	0

Since 1 is a zero,
$$x^3-7x+6=(x-1)(x^2+x-6)$$
$$x^2+x-6=(x+3)(x-2)$$
ANSWER: $\boldsymbol{(x-1)(x+3)(x-2)}$

22. An integer between 1 and 10000 inclusive is selected at random. What is the probability that it is a perfect square?
The probability of selecting a perfect square is equal to the number of perfect squares within the interval divided by the number of integers in that interval.
There are 100 perfect squares from 1 to 10 000.

ANSWER: $\boldsymbol{\frac{1}{100}}$ **or 0.01**

23. If a chord 24 cm long is 5 cm from the center of a circle, how long is a chord 10 cm from the center?
Let $\boldsymbol{c}$ be the chord length, $\boldsymbol{d}$ be the perpendicular distance from the center to the chord; and $\boldsymbol{r}$ be the radius.



By Pythagorean theorem,
$$r^2=\left(\frac{1}{2}c\right)^2+d^2$$
Given that $c=24, d=5$
Compute for r.
$$r^2=\left(\frac{1}{2}(24)\right)^2+5^2$$
$$r^2=12^2+5^2\rightarrow r^2=169\rightarrow r=13$$
$$r^2=\left(\frac{1}{2}c\right)^2+d^2\rightarrow c=2\sqrt{r^2-d^2}$$

$$c = 2\sqrt{13^2 - 10^2} \rightarrow c = 2\sqrt{69}$$

ANSWER: $2\sqrt{69}\ cm$

24. Pipes are being stored in a pile with 25 pipes in the first layer, 24 in the second, and so on. If there are 12 layers, how many pipes does the pile contain?
The problem presents an arithmetic series with $a_1 = 25, d = -1$, and $n = 12$.

$$S_n = a_1 n + \frac{n}{2}(n-1)d$$

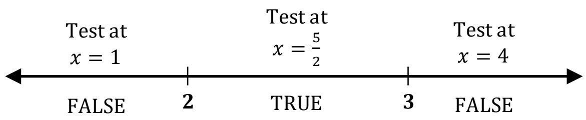
$$S_{12} = 25(12) + \frac{12}{2}(12-1)(-1) \rightarrow S_{12} = 234$$

ANSWER: 234

25. Find the solution set of the inequality $x^2 < 5x - 6$.

$$x^2 < 5x - 6$$
$$x^2 - 5x + 6 < 0$$
$$(x-3)(x-2) < 0$$

Split into three intervals to determine the domain of the inequality:
$(-\infty, 2), (2,3), (3, +\infty)$



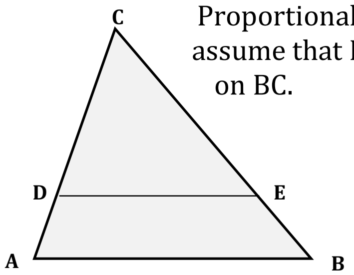
ANSWER: $2 < x < 3$

26. Suppose an object is dropped from the roof of a very tall building. After t seconds, its height from the ground is given by $h = -16t^2 + 625$, where h is measured in feet. How long does it take to reach ground level?

$$0 = -16t^2 + 625 \rightarrow t = \frac{25}{4}$$

ANSWER: $6.25\ s$

27. In ΔABC, $AB \parallel DE$, AC = 10 cm, CD = 7 cm, and CE = 9 cm. Find BC.



In order to apply the Triangle Proportionality Theorem, assume that D is on AC and E is on BC.

$$\frac{CE}{CD} = \frac{BC}{AC}$$

$$\frac{9}{7} = \frac{BC}{10}$$

ANSWER: $\frac{90}{7}$ cm or 12.86 cm

28. Find a polynomial $P(x)$ of degree 3 that has zeroes of -2, 0, and 7, and the coefficient of x^2 is 10.

$$P(x) = ax^3 + 10x^2 + cx + d$$
$$bx(x+2)(x-7) = bx^3 - 5bx^2 - 14bx$$
$$-5bx^2 = 10x^2$$
$$\frac{-5bx^2}{-5x^2} = \frac{10x^2}{-5x^2} \rightarrow b = -2$$
$$bx^3 - 5bx^2 - 14bx = -2x^3 + 10x^2 + 28x$$

ANSWER: $P(x) = -2x^3 + 10x^2 + 28x$

29. Find the equation (in the form $ax + by = c$) of the perpendicular bisector of the line segment joining the points (1, 4) and (7, -2).

A perpendicular bisector of a segment is perpendicular to the segment at its midpoint.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \rightarrow m = \frac{-2-4}{7-1} = -1$$

The slope of the perpendicular bisector is 1.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \rightarrow M = \left(\frac{1+7}{2}, \frac{4-2}{2}\right)$$

The perpendicular bisector should pass through the point (4, 1).

$$y_1 = mx_1 + b \rightarrow 1 = 1(4) + b \rightarrow b = -3$$
$$y = x - 3$$

ANSWER: $x - y = 3$

30. The sides of a polygon are 3, 4, 5, 6, and 7cm. Find the perimeter of a similar polygon, if the side corresponding to 5 cm is 9 cm long.

By similarity, $\frac{P_1}{5\ cm} = \frac{P_2}{9\ cm} \rightarrow \frac{3+4+5+6+7}{5\ cm} = \frac{P_2}{9\ cm}$

$$\frac{25\ cm}{5\ cm} = \frac{P_2}{9\ cm} \rightarrow P_2 = 45\ cm$$

ANSWER: $45\ cm$

31. If $\tan\theta = \frac{2}{3}$ and θ is in Quadrant III, find $\cos\theta$.
$\tan\theta = \frac{a}{b} = \frac{2}{3} \rightarrow a = -2$ and $b = -3$ since θ is in Quadrant III.

$$\cos\theta = \frac{b}{c}$$

Solve for c, through the Pythagorean theorem,

$$c^2 = a^2 + b^2 \to c^2 = (-2)^2 + (-3)^2 \to c = \sqrt{13}$$

$$\cos\theta = -\frac{3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

ANSWER: $-\frac{3\sqrt{13}}{13}$

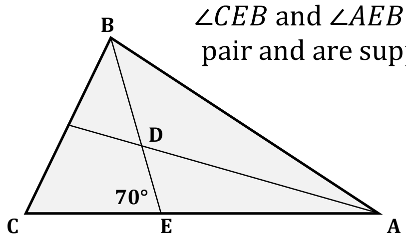
32. The first term of a geometric sequence is 1536, and the common ratio is $\frac{1}{2}$. Which term of the sequence is 6?

$$a_1 = 1536, r = \frac{1}{2}, a_n = 6, n = ?$$
$$a_n = a_1 r^{n-1}$$
$$6 = (1536)\left(\frac{1}{2}\right)^{n-1}$$
$$\frac{6}{1536} = \left(\frac{1}{2}\right)^{n-1}$$
$$\frac{1}{256} = \frac{1}{2^{n-1}}$$

$$2^{n-1} = 256 \to 2^{n-1} = 2^8$$
$$n - 1 = 8 \to n = 9$$

ANSWER: 9th term

33. In ΔABC; $\angle B = 2\angle A$. The bisectors of these angles meet at D, while BD extended meet AC at E. If $\angle CEB = 70°$, find $\angle BAC$.



$\angle CEB$ and $\angle AEB$ form a linear pair and are supplementary.

$$m\angle AEB = 110°$$
Since $m\angle BAC = m\angle EBA$ and
$$m\angle AEB + m\angle BAC + m\angle EBA = 180°$$
$$110° + 2\,m\angle BAC = 180°$$
$$m\angle BAC = 35°$$

ANSWER: 35°

34. Find the sum of the series:

$$\frac{1}{3} + \frac{2}{3^2} + \frac{2^2}{3^3} + \frac{2^3}{3^4} + \cdots$$

The infinite sequence described in the series is geometric in nature.

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_n = \frac{1}{3}\left(\frac{1 - \left(\frac{2}{3}\right)^n}{1 - \frac{2}{3}}\right) \to S_n = \frac{1}{3}\left(\frac{1 - \left(\frac{2}{3}\right)^n}{\frac{1}{3}}\right)$$

$$S_n = 1 - \left(\frac{2}{3}\right)^n \to S_n = 1 - \frac{2^n}{3^n}$$

$\frac{2^n}{3^n}$ is a quantity that approaching zero as n gets relatively smaller.

$$S_n = 1 - 0 \to S_n = 1$$

ANSWER: 1

35. What is the remainder when $x^3 - x + 1$ is divided by $2x - 1$?

By the remainder theorem,

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^3 - \frac{1}{2} + 1 \to f\left(\frac{1}{2}\right) = \frac{5}{8}$$

ANSWER: $\frac{5}{8}$

36. Triangle ABC has right angle at B. If $\angle C = 30°$ and BC = 12 cm, find AB.

ΔABC is a special right triangle. In a 30-60 right triangle, the side opposite the 60-degree angle is $\sqrt{3}$ times the length of the side opposite the 30-degree angle. Thus,

$$12 = \mathrm{AB}\sqrt{3} \to AB = 4\sqrt{3}$$

ANSWER: $4\sqrt{3}$

37. What is the largest root of the equation $2x^3 + x^2 - 13x + 6 = 0$?

Use the Rational zeroes theorem to try out possible zeroes.

2	2	1	-13	6
		4	10	-6
	2	5	-3	0

$$2x^3 + x^2 - 13x + 6 = 0$$
$$(x - 2)(2x^2 + 5x - 3) = 0$$
$$(x - 2)(2x - 1)(x + 3) = 0$$

The zeroes of the equation are $\mathbf{2}, \frac{\mathbf{1}}{\mathbf{2}}, \mathbf{-3}$

ANSWER: 2

38. Find the equation (in the form $x^2 + y^2 + cx + dy = e$) of the circle that has the points (1, 8) and (5,-6) as the endpoints of the diameter.

The center is the midpoint of the two given points.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \to M = \left(\frac{1+5}{2}, \frac{8-6}{2}\right)$$
$$M = (3,1)$$

The radius is the distance between one of the endpoints and the midpoint.

$$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$r = \sqrt{(5-3)^2 + (8-1)^2} \to r = \sqrt{53}$$

The equation of a circle whose center (h, k) is not at the origin may be obtained through

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-3)^2 + (y-1)^2 = 53$$
$$x^2 - 6x + 9 + y^2 - 2y + 1 = 53$$
$$x^2 + y^2 - 6x - 2y = 43$$

ANSWER: $\mathbf{x^2 + y^2 - 6x - 2y = 43}$

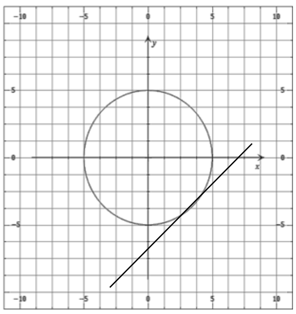
39. The distance from the midpoint of a chord 10 cm long to the midpoint of its minor arc is 3 cm. What is the radius of the circle?

Refer to item #23 for a similar example.

$$c = 2\sqrt{r^2 - d^2}$$
$$10 = 2\sqrt{r^2 - (r-3)^2}$$
$$10 = 2\sqrt{r^2 - (r^2 - 6r + 9)}$$
$$10 = 2\sqrt{6r - 9}$$
$$5 = \sqrt{6r - 9}$$
$$25 = 6r - 9 \to r = \frac{17}{3}$$

ANSWER: $\mathbf{\frac{17}{3}}$ **cm**

40. Find the equation (in the form $ax + by = c$) of the line tangent to the circle $x^2 + y^2 = 25$ at the point (3,-4).



The slope of the line containing the radius whose endpoints are the center and the given point must be the negative reciprocal of the slope of the tangent line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} \to m = \frac{-4-0}{3-0} = -\frac{4}{3} \to m_p = \frac{3}{4}$$
$$y - y_1 = m(x - x_1)$$
$$y + 4 = \frac{3}{4}(x-3) \to 3x - 4y = 25$$

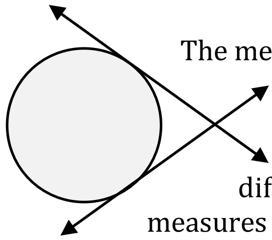
ANSWER: $\mathbf{3x - 4y = 25}$

41. How many triangles can be formed by 8 points, no three of which are collinear?

$$\frac{(C_3^8)}{2} = \frac{\frac{8!}{3!\,(8-3)!}}{2} = 56$$

ANSWER: 56

42. Two tangents to a circle form an angle of 80°. How many degrees is the larger intercepted arc?



The measure of the angle formed outside by two tangents to a circle is half the difference of the measures of the intercepted arcs.

$$m\angle A = \frac{1}{2}\left(m\widehat{BDC} - m\widehat{BC}\right)$$

Note however that $m\widehat{BDC} + m\widehat{BC} = 360°$

Therefore, $m\widehat{BC} = 360° - m\widehat{BDC}$

$$m\angle A = \frac{1}{2}\left(m\widehat{BDC} - 360° + m\widehat{BDC}\right)$$
$$80° = \frac{1}{2}\left(2\,m\widehat{BDC} - 360°\right)$$
$$160° = \left(2\,m\widehat{BDC} - 360°\right)$$
$$160° + 360° = 2\,m\widehat{BDC}$$
$$m\widehat{BDC} = 260°$$

ANSWER: $\mathbf{260°}$

43. Find all real solutions of the equation $x^{\frac{1}{3}} + x^{\frac{1}{6}} - 2 = 0$.

Treat the equation like a quadratic trinomial,

$\left(x^{\frac{1}{6}}\right)^2 + \left(x^{\frac{1}{6}}\right) - 2 = 0$ where $x^{\frac{1}{6}}$ is the variable.

Factor out as,

$$\left(x^{\frac{1}{6}} + 2\right)\left(x^{\frac{1}{6}} - 1\right) = 0$$

Solve for x.

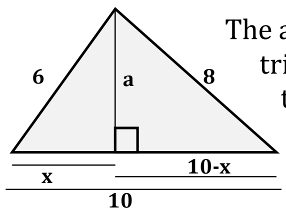
$$x^{\frac{1}{6}} + 2 = 0 \to x = 64$$

$$x^{\frac{1}{6}} - 1 = 0 \rightarrow x = 1$$

The equation does not hold true for $x = 64$, this being an extraneous root.

ANSWER: 1

44. The sides of a triangle are 6, 8, and 10, cm, respectively. What is the length of its shortest altitude?



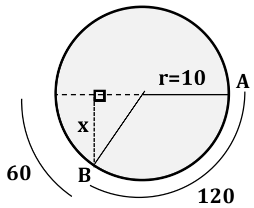
The altitude to a side of a triangle divides the triangle into two similar triangles. The shortest altitude is that to the longest side.

By proportion, $\frac{x}{a} = \frac{6}{8} = \frac{a}{10-x}$

$$x = \frac{3a}{4}, \qquad \frac{3}{4} = \frac{a}{10 - \frac{3a}{4}} \rightarrow a = \frac{24}{5} \text{ or } 4.8$$

ANSWER: $\boldsymbol{\frac{24}{5}}$ ***or* 4.8 *cm***

45. In a circle of radius 10 cm, arc AB measures 120°. Find the distance from B to the diameter through A.



The triangle is special

The side opposite the 30-degree angle is 5 cm because it is half the hypotenuse which happens to be a radius.

Therefore, the length of *x* is $5\sqrt{3}$

ANSWER: $\boldsymbol{5\sqrt{3}}$ ***cm***

46. Find the remainder when x^{100} is divided by $x^2 - x$.

$$\frac{x^{100}}{x^2 - x} = \frac{x^{100}}{x(x-1)} = \frac{x}{x} \cdot \frac{x^{99}}{x-1}$$

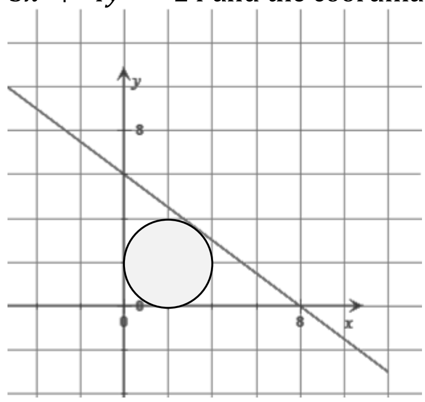
By the remainder theorem,

$$P(1) = 1^{99} = 1$$

Multiplying the remainder by *x*, the remainder becomes *x.*

ANSWER: $\boldsymbol{x}$

47. Find the area (in terms of π) of the circle inscribed in the triangle enclosed by the line $3x + 4y = 24$ and the coordinate axes.



$$r = \frac{A_t}{s}$$

Where r is the radius of the circle, A_t is the area of the triangle, and s is the semi-perimeter ($s = \frac{1}{2}(a + b + c)$)

$$A_c = \pi r^2, A_t = \frac{1}{2} bh$$

$$A_c = \pi \left(\frac{\frac{1}{2} bh}{s} \right)^2$$

$$A_c = \pi \left(\frac{\frac{1}{2}(6)(8)}{\frac{1}{2}(6 + 8 + 10)} \right)^2$$

$$A_c = \pi \left(\frac{24}{12} \right)^2 \rightarrow A_c = 4\pi$$

ANSWER: $\boldsymbol{4\pi\ cm^2}$

48. In how many different ways can 5 persons be seated in an automobile having places for 2 in the front seat and 3 in the back if only 2 of them can drive and one of the others insists on riding in the back?

<table>
<tr><td>(2) Driver's seat
2 possibilities</td><td colspan="2">(1) The last person</td></tr>
<tr><td>(1) The person insisting to sit on the back</td><td>(3) Any of the three others</td><td>(2) Any of the two left</td></tr>
</table>

By Fundamental Principle of Counting,

$$2 \cdot 1 \cdot 1 \cdot 3 \cdot 2 = 12$$

The three passengers at the back may be rearranged in three different ways, so,

$$12 \cdot 3 = 36$$

ANSWER: 36

49. Factor completely the expression $x^4 + 3x^2 + 4$.

$$x^4 + 3x^2 + 4 \rightarrow x^4 + 4x^2 + 4 - x^2$$
$$(x^4 + 4x^2 + 4) - x^2$$
$$(x^2 - 4)^2 - x^2$$
$$[(x^2 - 4) - x][(x^2 - 4) + x]$$
$$(x^2 - x - 4)(x^2 + x - 4)$$

ANSWER: $\boldsymbol{(x^2 - x - 4)(x^2 + x - 4)}$

50. A bag is filled with red and blue balls. Before drawing a ball, there is a $\frac{1}{4}$ chance of drawing a blue ball. After drawing out a ball, there is now a $\frac{1}{5}$ chance of drawing a blue ball. How many red balls are originally in the bag?

Since $P(b_2) < P(b_1)$, the ball first drawn is blue.

$$P(b_1) = \frac{b}{b+r} = \frac{1}{4}$$
$$P(b_1) = \frac{b-1}{b-1+r} = \frac{1}{5}$$
$$\frac{b}{b+r} = \frac{1}{4} \rightarrow 4b = b + r \rightarrow r = 3b$$
$$\frac{b-1}{b-1+r} = \frac{1}{5} \rightarrow \frac{b-1}{b-1+3b} = \frac{1}{5}$$
$$\frac{b-1}{4b-1} = \frac{1}{5} \rightarrow 5b - 5 = 4b - 1$$
$$b = 4; r = 12$$

ANSWER: 12