Metrobank-MTAP-DepEd Math Challenge 2016 Elimination round

Answers with Solutions

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10. The sum of the measures of the interior angles of a polygon is 1980°. How many sides has the polygon?
11. The average of three numbers is 20. Two numbers are added to the set and the average of the five numbers becomes 42. If one of the added numbers is twice the other, what are the two numbers added to the data set?
12. Compute $24 \div \frac{1+\frac{1}{5}}{2-\frac{1}{3}}$.
13. Subtract $5a - 2b + c$ from the sum of $3a + b - 2c$ and $a - b + 3c$.
14. Alex, Beth, and Carla play a game in which the losing player in each round gives each of the other players as much money as the player has at that time. In Round 1, Alex loses and gives Beth and Carla as much money as they have. In round 2, Beth loses and in Round 3, Carla loses. After 3 rounds, they find that they each have Php 40. How much money did Alex have at the start of the game?
15. Three two-digit numbers have consecutive tens digits and have units digit all equal to 5. If the tens digit of the smallest number is n, what is the sum of the three numbers?
16. The length of a rectangle is 8 cm more than its width. If the length is decreased by 9 and the width is tripled, the area is increased by 50%. What was the area of the original rectangle?

17. Evaluate: $236 \times 542 + 458 \times 764 + 542 \times 764 + 236 \times 458$.

18. TRUE or FALSE: If n is a real number, then n^2 is positive.

19. If n kilos of rice costs p pesos, how much will x kilos of rice cost?

20. If the letters of the word MATHEMATICS are repeatedly and consecutively written, what is the 2016th letter?

21. Simplify: $\dfrac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x^3-2x}$.

22. If x is three times as far from -5 as it is from 15, what are the possible values of x?

23. If three children eat 4 kilos of rice in 5 days, how long will 12 children eat 48 kilos of rice?

24. A conical tank full of water is emptied into an empty cylindrical tank of the same height. If the base radius of the cylinder is twice that of the cone, what fraction of the cylinder will be filled with water?

25. Find the minimum integer n for which $\dfrac{18}{n+1}$ is an integer.

26. The sum of the square roots of two positive integers is 5. If the two integers differ by 5, what are the integers?

27. A worm crawls 7.5 inches in 80 seconds. What is its speed in feet per hour?

28. By how much is $(3x-5)(x+2)$ greater than $(x+4)(2x-1)$?

29. A game consists of drawing a number from $1-20$. A player wins if the number drawn is either a prime number or a perfect square. What is the probability of winning in this game?

30. The number of boys in a class is equal to the number of girls. Nine boys are absent today, and this leaves twice as many girls as boys in the classroom. How many students belong to the class?

31. A square region is removed from a rectangular region. Which of the following can be true?

    (a) The perimeter is decreased.

    (b) The perimeter is not changed.

    (c) The perimeter is increased.

32. A bag contains 5 black, 5 red, 6 blue, 7 white, 8 yellow, and 10 orange beads. At least how many beads must be drawn from the bag to ensure that at least 3 beads of the same color are chosen?

33. How many positive numbers less than 1000 are divisible by 6 but not by 5?

34. A $4\,\text{cm} \times 5\,\text{cm} \times 7\,\text{cm}$ rectangular prism is painted on all faces. If the prism is cut into $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$ how many cubes do not have paint on any of its sides?

35. Andrew is 5 years old and Charlie is 26. In how many years will Charlie be $2\frac{1}{2}$ times as old as Andrew?

36. A car is driving along a highway at 55 kph. The driver notices a bus, $\frac{1}{2}$ km behind. The bus passes the car one minute later. What was the speed of the bus?

37. With what polynomial must $8x^5 - 10x^3 + 2x + 5$ be divided to get a quotient of $4x^2 - 3$ and a remainder of $5 - x$?

38. The volume of a sphere is equal to its surface area. What is the diameter of the sphere?

39. If $\overline{3A54B10}$ is divisible by 330, what are the values of A and B?

40. Find the solution set: $|5 - 2x| < 19$.

41. If $\overline{47A2969}$ is the square of $3(721 + A)$, find the digit A.

42. Consider the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots$. How many numbers in the sequence are needed so that the sum of the reciprocals is 100?

43. If n is a positive odd integer, which of the following is a perfect square: $2^{n^2}, 25(3^{n+2}), 7^{n(n+1)}$?

44. The sum of the first 50 odd integers is 2500. What is the sum of the next 50 odd integers?

45. Felix has an average of 90 in five tests, each test is with 100 points. What is the lowest possible score Felix could have gotten in a test?

46. In square $ABCD$, P is the midpoint of AB and Q is the midpoint of BC. What percent of the area of $ABCD$ is the area of $\triangle PQD$?

47. If $1 < x < \frac{9}{8}$, which is bigger, $\sqrt[3]{3x}$ or $\sqrt{2x}$?

48. Find the least positive integer that leaves remainders of 1, 2, and 3 when divided by 3, 5, and 7, repsectively.

49. Find all points x on the real number line such that the sum of the distances from x to 4 and from x to -4 is 12.

50. If $11n$ leaves a remainder of 6 when divided by 7, what is the remainder when $5n$ is divided by 7?

1.2 Grade 8

1. Simplify $(a-b)^2(a+b)^2+2a^2b^2$.

2. Simplify $\left(\frac{125x^4y^3}{27x^{-2}y^6}\right)^{\frac{1}{3}}$.

3. Solve for x in the equation $x^4-5x^2+4=0$.

4. In the arithmetic sequence $10+10\sqrt{3}, 11+9\sqrt{3}, 12+8\sqrt{3}, \cdots$, what term has no $\sqrt{3}$?

5. If $x+y=12$ and $xy=50$, what is x^2+y^2?

6. What is the sum of the first ten terms of the geometric sequence 4, 8, 16, $\cdots$,?

7. If the product of two consecutive odd integers is 783, what is the sum of their squares?

8. If r and s are the roots of the equation $2x^2-3x+4=0$, what is $4r^2+7rs+4s^2$?

9. A long wire is cut into three smaller pieces in the ratio of $7:3:2$. If the shortest piece is 16 cm, what is the area of the largest rectangle that can be created using the longest piece?

10. A boat takes $\frac{2}{3}$ as much time to travel downstream as to its return. If the rate of the river's current is 8 kph, what is the rate of the boat in still water?

11. How many prime numbers between 40 and 240 ends with 4?

12. Simplify: $x(1-6x)-(1-2x)(3x-2)$.

13. What is the last digit if 7^{2016}?

14. If $a=3$ and $b=7$, what is $4a^3b+6a^2b^2+4ab^3$?

15. What is the median of the numbers $a+1, a+3, a-2, a+5$, and $a-4$?

16. A rectangle is formed by putting two squares side by side. If each square has perimeter 28 cm, what is the periemter of the rectangle?

17. Solve for x: $4(1-3x)-2x(1-3x)+5(1-3x)+3x(1-3x)=0$.

18. A jacket was worth Php 1200. hoping to gain more profit, the shop owner increased its price by 10 %, but was later forced to reduce it by 15 % since there were no takers. What was the final price of the jacket?

19. Let $\{a_n\}$ be an arithmetic sequence. If $a_4=27$ and $a_9=67$, what is a_1?

20. Triangle ABC is isosceles with $AB=AC$. Let D be the foot of the altitude from A on BC, and let E be the point on side AC such that DE bisects $\angle ADC$. If $\angle DEC=67°$, what is $\angle BAC$?

21. What is the greatest integer less than or equal to $\left(2+\sqrt{3}\right)^2$?

22. Each week, a pet owner buys m kilograms of bananas for its monkeys. If each mokey eats n kilograms of bananas each day, how many monkeys does the owner have? Express your answer in terms of m and n.

23. If $8.07^3 = 525.557943$, what is 0.807^3?

24. Solve for x: $3 < |1+2x|$.

25. What is the value of $\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}$?

26. Liza ran 200 meters in only 45 seconds. What was Lisa's speed in kilometers per hour?

27. Suppose $f(x)$ is a linear function such that $f\left(-\frac{1}{2}\right) = -7$ and $f(1) = -3$. What is $f(-3)$?

28. Anna has four cardboard squares, each of which has side of length 6 cm. She decides to form a trapezoid by putting three squares side-by-side, cutting one square along a diagonal, discarding one-half and putting the other half at one end of the squares. What is the area of the trapezoid?

29. What is the perimeter of the trapezoid in # 28?

30. The number 9979 is a four-digit number the sum of whose digits is equal to 34. How many such numbers exist?

31. If $\underbrace{10^{10}+10^{10}+\cdots+10^{10}}_{10\,terms} = 10^x$, what is x?

32. If the sum of the reciprocals of the roots of the equation $3x^2+7x+k$ is $-\frac{7}{3}$, what is k?

33. If $x = 1+\sqrt{3}$, find the value of $\frac{2x^2-4x+8}{3x^2-6x+10}$.

34. The sum of the angles of a regular polygon is 2160°. How many sides does it have?

35. An iron cube of side 16 cm is melted down and is used to make eight smaller iron cubes of the same size. What is the length of the sides of the smaller iron cube?

36. The sum of two numbers is 20. If one of the numbers is three times the other, what is their product?

37. Determine how many ordered pairs (x, y) satisfy the system

$$\begin{aligned} x^2+y^2 &= 25 \\ x-y &= 5. \end{aligned}$$

38. A 400mL flask containing 40 % alcohol mixture and a 600mL flask containing a 60 % alcohol mixture are put together in a single large flask. How many percent alcohol is the resulting mixture?

39. For what values of k will the equation $y = x^2 + kx + k$ cross the x-axis twice?

40. A certain farmer only raises chickens and pigs. Altogether, the animals have 57 heads and 158 legs. How many chickens does he have?

41. Solve for x: $|x - 4| = |2x + 1|$

42. The sum of 100 numbers is 34287. The teacher writes all the numbers on the board and proceed as follows: he adds 1 to the first number, 2 to the second number, 3 to the third, and so on, and adds 100 to the last. What is the sum of the new set of numbers?

43. Express the answer in lowest terms $\dfrac{6a-2}{9a} \cdot \dfrac{9a^2}{24a-8}$.

44. The sum of the roots of the quadratic function $x^2 - 4x + 3$ is 4. If all the coefficients of the quadratic are increased by 2, what is the sum of the roots of the new function?

45. What is the constant term in the expansion of $\left(2x^2 + \dfrac{1}{2x}\right)^6$?

46. Suppose that a is a positive number such that the roots of $x^2 - ax + 1 = 0$ differ exactly by 1. What is the value of a?

47. Two positive integers are relatively prime if they have no comon factor other than 1. How many two-digit numbers are relatively prime with 24?

48. A motorist travelled a distance of 180 km. If he had driven 30km/h faster, he could have travelled the same distance 1 hour less time. How fast did he drive?

49. Solve for x in the inequality $5 < |1 - 2x| \leq 7$.

50. Joe draws a line on the board and marks five poionts on that line. He then marks two points on the board that lies on the same line, obtaining a total of seven points. How many different triangles can be drawn using the seven points as vertices?

1.3 Grade 9

1. Solve for x in $\sqrt{2-x}-4=0$.
2. Find $\sin\theta$ if $\tan\theta=\dfrac{3}{4}$ and $\cos\theta<0$.
3. What is the value of $\tan\left(-\dfrac{17}{6}\pi\right)$?
4. Sovle for x in $\dfrac{5}{x+2}-\dfrac{2}{x+1}=\dfrac{1}{2}$.
5. Solve for x in $x^4-4x^2-5=0$.
6. Find the sum of the first 20 odd numbers.
7. Solve for x in $x^2+12<7x$.
8. Find all possible values of x if one of the interior angles of a square is $(2x^2-8x)^\circ$.
9. What is the sum of the interior angles of a pentagon?
10. The supplement of an angle is four times its complement. Find the angle.
11. What is the shortest side of $\triangle ABC$ if $\angle P=57^\circ$ and $\angle Q=65^\circ$?
12. Rationalize the denominator and simplify: $\dfrac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.
13. Find the 25th term of the arithmetic sequence whose first 3 terms are 4,7, and 10.
14. Solve for x in $\sqrt{2x+1}-\sqrt{x}=1$.
15. For what value/s of x is $x^2-8x+18\geq 0$?
16. In $\triangle ABC, \angle B=90^\circ$ and $\sin A=\dfrac{3}{4}$. Determine the value of $\tan C$.
17. The angle bisector at vertex B of $\triangle ABC$ meets AC at D. If $BC=6, AC=40$ and $CD=15$, find AB.
18. Find the equation of the line parallel to $4x+3y=12$ and passing through $(-12,4)$.
19. Express in terms of sines and cosines of θ and simplify: $\cot\theta\sec^2\theta$.
20. Find the common ratio of a geometric sequence whose first term is -2 and the 7th term is -1458.
21. Solve for x in $4\sqrt{3x+1}=4x+3$.
22. Find the 7th term of the geometric sequence: 8, 12, 18, $\cdots$.

23. Triangle ABC is a right triangle with $C = 90°$. If $A = 60°$ and $a = 50$, find b.

24. Two of the exterior angles of a regular polygon have measures $(6x - 30)°$ and $(114 - 10x)°$. How many sides does this regular polygon have?

25. Solve for x in $\dfrac{3}{2x-1} - \dfrac{2}{x} = 0$.

26. Find x so that $x - 2, x + 2$ and $x + 4$ are consecutive terms of a geometric seuqnece.

27. What is the smallest positive angle which is co-terminal to $-1125°$?

28. What is the height of an equilateral triangle whose perimeter is 6 meters?

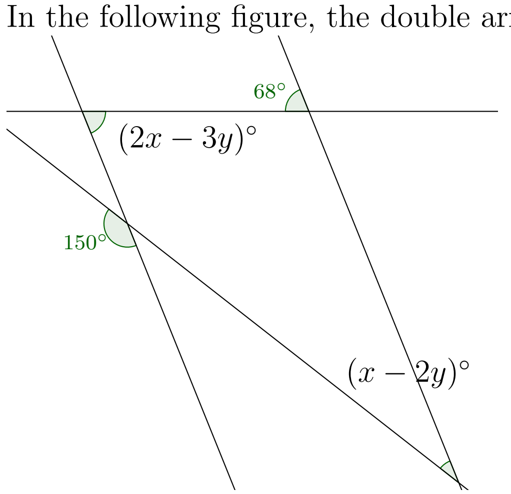
29. By what factor is the volume of a cube increased if each of its sides is tripled?

30. z varies directly as x and varies ivnersely as the square of y. If $z = \dfrac{7}{2}$ when $x = 14$ and $y = 6$, find z when $x = 27$ and $y = 9$.

31. Express in terms of sines or cosines of θ and simplify: $\dfrac{\cot^2\theta + 1}{\tan^2\theta + 1}$.

32. Right $\triangle ABC$, with right angle at C, has sides $b = 5$ and $c = 7$. Find $\csc B$.

33. In the following figure, the double arrows indicate parallel lines. Find x.

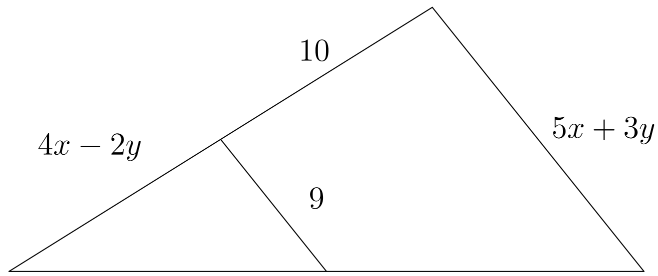


34. What is the perimeter of an equilateral triangle whose area is $75\sqrt{3}$ sq. cm?

35. A person standing 40 ft away from a street light that is 25 ft tall. How tall is he if his shadow is 10 ft long?

36. What is the maximum value of $f(x) = -2x^2 - 4x + 3$?

37. The figure shows a segment joining the midpoints of two sides of a triangle. What is the sum of x and y?



38. If $x > 1$, is $\frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$ positive or negative?

39. The diagonals of a rhombus are in the ratio of 1:3. If each side of the rhombus is 10 cm long, find the length of the longer diagonal.

40. Find a and b so that the zeroes of $ax^2 + bx + 24$ are 3 and 4.

41. Find all k so that the graph of $y = -\frac{1}{4}x^2 + kx - 9$ is tangent to the x-axis.

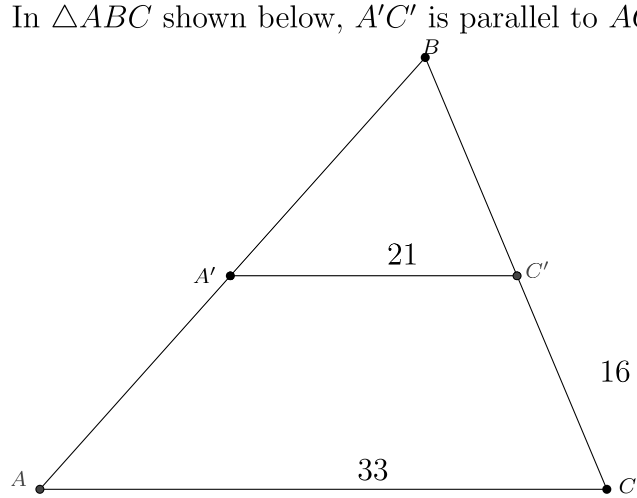
42. The diagonals of parallelogram $JKLM$ intersect at P. If $PM = 3x - 2, PK = x + 3$ and $PJ = 4x - 3$, find the length of PL.

43. Suppose that w varies directly as x and the square of y and inversely as the square root of z. If x is increased by 80 %, y is increased by 40 %, and z is increased by 44 %, by how many percent will w increase?

44. Find k so that the minimum value of $f(x) = x^2 + kx + 8$ is equal to the maximum value of $g(x) = 1 + 4x - 2x^2$.

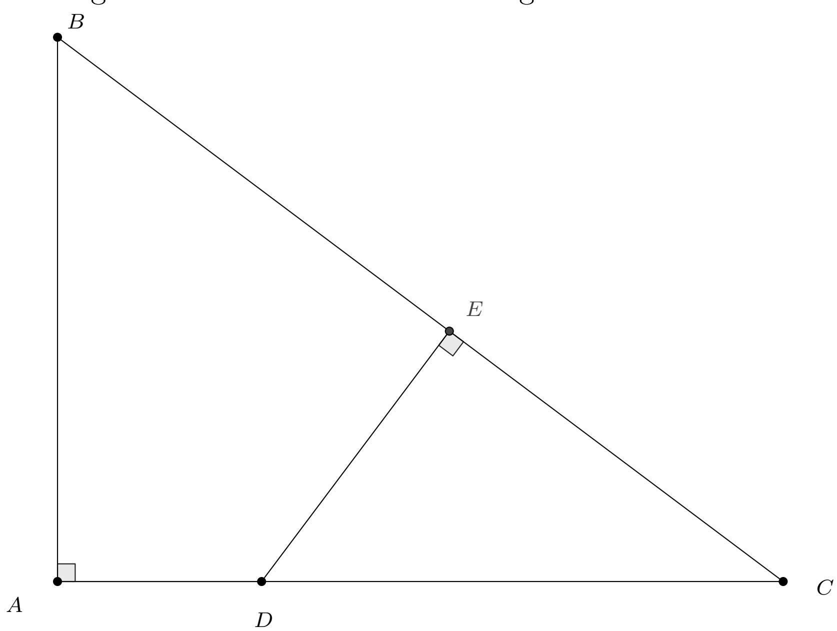
45. The difference of two numbers is 22. Find the numbers so that their product is to be minimum.

46. In $\triangle ABC$ shown below, $A'C'$ is parallel to AC. Find the length of BC'.



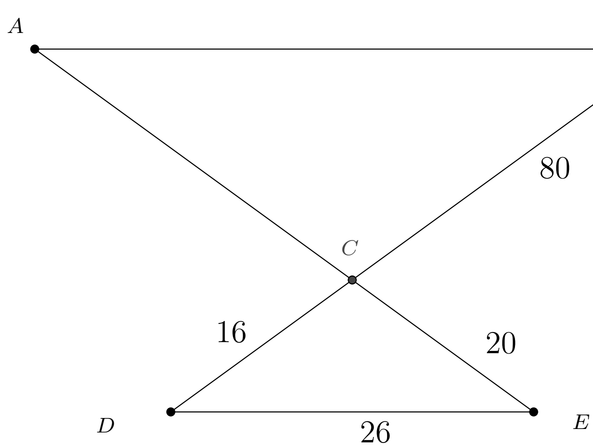
47. Find the length of h, the height drawn to the hypotenuse, of the right $\triangle ABC$ with right angle at C if h divides the hypotenuse into two parts of length 25 (from A) and 9.

48. In the figure below, $\angle BAC$ and $\angle DEC$ are both right angles, $CD = 6, BC = 10$, and the length of AD is one-fourth the length of AC. Find CE.



49. The number r varies jointly as s and the square of t. IF $r = 6$ when $s = 12$ and $t = \dfrac{1}{2}$, find r when $s = 18$ and $t = \dfrac{3}{2}$.

50. Given the figure below with AB parallel to DE. Find the length of AB.



1.4 Grade 10

1. Find the value of $\sqrt{x+\sqrt{14-x}}$ when $x=-2$.

2. Find the area of the triangle formed by the coordinate axes and the line $3x+2y=6$.

3. A sequence is defined by $a_n = 3(a_{n-1}+2)$ for $n \geq 2$, where $a_1 = 1$. What is a_4?

4. Christa leaves Town A. After traveling 12 km, she reaches Town B at 2:00 P.M. Then she drove at a constant speed and passes Town C, 40 km from Town B, at 2:50 P.M. Find the function $d(t)$ that models the distance (in km) she has traveled from Town A t minutes after 2:00 P.M.

5. What is the largest negative integer that satisfies the inequality $|3x+2| > 4$?

6. A person has two parents, four grandparents, eight grand-parents, and so on. How many ancestors does a person have 10 generations back?

7. In $\triangle ABC$, $\angle B$ is twice $\angle A$, and $\angle C$ is three times as large as $\angle B$. Find $\angle C$.

8. If $-3 \leq x \leq 0$, find the minimum value of $f(x) = x^2 + 4x$.

9. The 9th and the 11th term of an arithmetic sequence are 28 and 45, respectively. What is the 12th term?

10. Perform the indicated operation, and simplify: $\dfrac{2}{x} + \dfrac{3}{x-1} - \dfrac{4}{x^2-x}$.

11. Find the range of the function $f(x) = |2x+1|$.

12. Find the value of $\left[x - (x - x^{-1})^{-1}\right]^{-1}$ when $x = 2$.

13. Find the equation (in the form $ax + by = c$) of the line through the point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.

14. In $\triangle ABC$, $\angle B = 90°$, $\angle ACB = 28°$, and D is the midpoint of AC. What is $\angle BDC$?

15. Find all solutions of the system: $\begin{cases} 2x + y = 1 \\ 3x + 4y = 14 \end{cases}$.

16. If $f(2x-1) = x$, what is $f(2)$?

17. What is the quotient when $3x^5 + 5x^4 - 4x^3 + 7x + 3$ is divided by $x + 2$?

18. A man is walking away from a lamppost with a light source 6 meters above the ground. The man is 2 meters tall. How long is his shadow when he is 110 meters from the lamppost?

19. Find the length of the shorter segment made on side AB of $\triangle ABC$ by the bisector of $\angle C$, if $AB = 20$, $AC = 12$, and $BC = 18$ cm.

20. How many different three-digit numbers less than 300 can be formed with the digits 1,2,3 and 5 if repetition of digits is not allowed?

21. Factor completely the expression $x^3 - 7x + 6$.

22. An integer between 1 and 10000 inclusive is selected at random. What is the probability that it is a perfect square?

23. If a chord 24 cm long is 5 cm from the center of a circle, how long is a chord 10 cm from the center?

24. Pipes are being stored in a pile with 25 pipes in the first layer, 24 in the second, and so on. If there are 12 layers, how many pipes does the pile contain?

25. Find the solution set of the inequality $x^2 < 5x - 6$.

26. Suppose an object is dropped from the roof of a very tall building. After t seconds, its height from the ground is given by $h = -16t^2 + 625$, where h is measured in feet. How long does it take to reach ground level?

27. In $\triangle ABC, AB || DE, AC = 10$ cm, $CD = 7$ cm, and $CE = 9$ cm. Find BC.

28. Find a polynomial $P(x)$ of degree 3 that has zeroes -2, 0, and 7, and the coefficient of x^2 is 10.

29. Find the equation (in the form $ax + by = c$) of the perpendicular bisector of the line segment joining the points $(1, 4)$ and $(7, -2)$.

30. The sides of a polygon are 3,4,5,6, and 7cm. Find the perimeter of a similar polygon, if the side corresponding to 5 cm is 9 cm long.

31. If $\tan\theta = \dfrac{2}{3}$ and θ is in Quadrant III, find $\cos\theta$.

32. The first term of a geometric sequence is 1536, and the common ratio is $\dfrac{1}{2}$. Which term of the sequence is 6?

33. Inf $\triangle ABC, \angle B = 2\angle A$. The bisectors of theses angles meet at D, while BD extended meet AC at E. If $\angle CEB = 70°$, find $\angle BAC$.

34. Find the sum of the series: $\dfrac{1}{3} + \dfrac{2^1}{3^2} + \dfrac{2^2}{3^3} + \dfrac{2^3}{3^4} + \cdots$.

35. What is the remainder when $x^3 - x + 1$ is divided by $2x - 1$?

36. Triangle ABC has right angle at B. If $\angle C = 30°$ and $BC = 12$ cm, find AB.

37. What is the largest root of the equation $2x^3 + x^2 - 13x + 6 = 0$?

38. Find the equation (in the form $x^2 + y^2 + cx + dy = e$) of the circle that has the points $(1, 8)$ and $(5, -6)$ as the endpoints of the diameter.

39. The distance from the midpoint of a chord 10 cm long to the midpoint of its minor arc is 3 cm. What is the radius of the circle?

40. Find the equation (in the form $ax+by=c$) of the line tangent to the circle $x^2+y^2=25$ at the point $(3,-4)$.

41. How many triangles can be formed by 8 points, no three of which are collinear?

42. Two tangents to a circle form an angle of 80°. How many degrees is the larger intercepted arc?

43. Find all real solutions of the equation $x^{\frac{1}{2}}+x^{\frac{1}{6}}-2=0$.

44. The sides of a triangle are 6, 8, and 10, cm, respectively. What is the length of its shortest altitude?

45. In a circle of radius 10 cm, arc AB measures 120°. Find the distance from B to the diameter through A.

46. Find the remainder when x^{100} is divided by x^2-x.

47. Find the area (in terms of π) of the circle inscribed in the triangle enclosed by the line $3x+4y=24$ and the coordinate axes.

48. In how many different ways can 5 persons be seated in an automobile having places for 2 in the front seat and 3 in the back if only 2 of them can drive and one of the others insists on riding in the back?

49. Factor completely the expression x^4+3x^2+4.

50. A bag is filled with red and blue balls. Before drawing a ball, there is a $\frac{1}{4}$ chance of drawing a blue ball. After drawing out a ball, there is now a $\frac{1}{5}$ chance of drawing a blue ball. How many red balls are originally in the bag?

2 Answers with Solutions

2.1 Grade 7

1. Simplify: $6(2)^2 - (4-5)^3$.
   Answer: 25

$$\begin{aligned} 6(2^2) - (4-5)^3 &= 6(4) - (-1)^3 \\ &= 24 - (-1) \\ &= \boxed{25} \end{aligned}$$

2. By how much is $3 - \frac{1}{3}$ greater than $\frac{1}{2} - 2$?
   Answer: $\mathbf{\frac{25}{6}}$

$$\begin{aligned} \left(3 - \frac{1}{3}\right) - \left(\frac{1}{2} - 2\right) &= \frac{8}{3} - \left(-\frac{3}{2}\right) \\ &= \boxed{\frac{25}{6}} \end{aligned}$$

3. Write $\frac{11}{250000}$ in scientific notation.
   Answer: $\mathbf{4.4 \times 10^{-7}}$

$$\text{Since } \frac{11}{250000} = \frac{11}{25} \cdot 10^{-4} \text{ and } \frac{11}{25} = 0.44,$$
$$\frac{11}{250000} = 0.44 \times 10^{-6} = \boxed{4.4 \times 10^{-7}}$$

4. The product of two prime numbers is 302. What is the sum of the two numbers?
   Answer: 153
   Since 302 is even, one of the factors is 2. Thus the other prime factor is 151. Which gives a sum of $\boxed{153}$.

5. A shirt is marked Php 315 after a discount of 10% and value added tax of 12%. What was the price of the shirt before the tax and discount?
   Answer: Php 312.5
   Let p be the original price of the shirt. So we have $0.9p$ after the 10 % discount and $(0.9)(1.12)p$ after the 12% added tax. Thus,

$$\begin{aligned} (0.9)(1.12)p &= 315 \\ \left(\frac{9}{10}\right)\left(\frac{28}{25}\right)p &= 315 \\ \left(\frac{126}{125}\right)p &= 315 \\ p &= \boxed{\text{Php}\,312.5} \end{aligned}$$

6. How many different lengths of diagonals does a regular octagon have?
Answer: 3
Draw an octagon and count the number of diagonals from a vertex with different lengths.

7. What number is midway between $3+\frac{1}{3}$ and $2-\frac{1}{3}$?
Answer: $\mathbf{\frac{5}{2}}$

$$\frac{\left(3+\frac{1}{3}\right)+\left(2-\frac{1}{3}\right)}{2}=\boxed{\frac{5}{2}}$$

8. Simplify: $(2-5)\times\left(-\frac{9}{8}\right)-\frac{3}{4}(-2)$.
Answer: $\mathbf{\frac{39}{8}}$

$$\begin{aligned}(2-5)\times\left(-\frac{9}{8}\right)-\frac{3}{4}(-2) &= (-3)\left(-\frac{9}{8}\right)+\frac{3}{2}\\ &= \frac{27}{8}+\frac{3}{2}\\ &= \boxed{\frac{39}{8}}\end{aligned}$$

9. Simplify: $\left(\frac{7}{2}+\frac{5}{6}\right)^2-\left(\frac{7}{2}-\frac{5}{6}\right)^2$.
Answer: $\mathbf{\frac{35}{8}}$
Note that $(a+b)^2-(a-b)^2=4ab$. Applying that identity to the problem,

$$\left(\frac{7}{2}+\frac{5}{6}\right)^2-\left(\frac{7}{2}-\frac{5}{6}\right)^2=4\left(\frac{7}{2}\right)\left(\frac{5}{6}\right)=\boxed{\frac{35}{3}}$$

10. The sum of the measures of the interior angles of a polygon is 1980°. How many sides has the polygon?
Answer: 13

$$\begin{aligned}180(n-2) &= 1980\\ n-2 &= 11\\ n &= \boxed{13}\end{aligned}$$

11. The average of three numbers is 20. Two numbers are added to the set and the average of the five numbers becomes 42. If one of the added numbers is twice the other, what

are the two numbers added to the data set?
Answer: 50 and 100
Let S be the sum of the three numbers. So $S = 3(20) = 60$. Let a and b be the two numbers added to the set. Now we have $S + a + b = 5(42) = 210$. Subtracting the two equations, $a + b = 150$. Also, $a = 2b$, gives $2b + b = 150$. Hence, $b = 50$. The numbers added are $\boxed{50 \text{ and } 100}$.

12. Compute $24 \div \frac{1+\frac{1}{5}}{2-\frac{1}{3}}$.
Answer: 12

$$\begin{aligned} 24 \div \frac{1+\dfrac{1}{5}}{2-\dfrac{1}{3}} &= 24 \div \frac{\dfrac{6}{5}}{\dfrac{5}{3}} \\ &= 24 \div 2 \\ &= \boxed{12} \end{aligned}$$

13. Subtract $5a - 2b + c$ from the sum of $3a + b - 2c$ and $a - b + 3c$.
Answer: $-\mathbf{a} + \mathbf{2b}$

$$\begin{aligned} [(3a+b-2c)+(a-b+3c)]-(5a-2b+c) &= (4a+c)-(5a-2b+c) \\ &= 4a+c-5a+2b-c \\ &= \boxed{-a+2b} \end{aligned}$$

14. Alex, Beth, and Carla play a game in which the losing player in each round gives each of the other players as much money as the player has at that time. In Round 1, Alex loses and gives Beth and Carla as much money as they have. In round 2, Beth loses and in Round 3, Carla loses. After 3 rounds, they find that they each have Php 40. How much money did Alex have at the start of the game?
Answer: 65
Let the starting amounts of Alex, Beth, and Carla be a, b and c.
The table below shows the amounts of they have every after round

Round	Alex	Beth	Carla
1	$a-b-c$	$2b$	$2c$
2	$2a-2b-2c$	$-a+3b-c$	$4c$
3	$4a-4b-4c$	$-2a+6b-2c$	$-a-b+7c$

Then,

$$4a-4b-4c=40 \rightarrow a-b-c = 10 \quad (1)$$
$$-2a+6b-2c=40 \rightarrow a-3b+c = -20 \quad (2)$$
$$-a-b+7c=40 \rightarrow a+b-7c = -40 \quad (3)$$

From (1), (2), and (3), $a = 65, b = 35, c = 20$. Thus, Alex had $\boxed{65}$ coins at the start of the game.

15. Three two-digit numbers have consecutive tens digits and have units digit all equal to 5. If the tens digit of the smallest number is n, what is the sum of the three numbers?
Answer: $\mathbf{30n + 45}$
So n is the smallest tens digit. Also, note that the integer $\overline{AB}$ can be written as $10A + B$. So the numbers are:
$10n + 5, 10(n + 1) + 5, 10(n + 2) + 5$ or $10n + 5, 10n + 15, 10n + 25$
Hence, their sum is $\boxed{30n + 45}$.

16. The length of a rectangle is 8 cm more than its width. If the length is decreased by 9 and the width is tripled, the area is increased by 50%. What was the area of the original rectangle?
Answer: 180 $\mathbf{cm^2}$
Let w be the width of the rectangle. So the length is $w+8$ and the area of the rectangle is $w(w + 8)$. Using the first condition,

$$\begin{aligned} [(w+8)-9]\,(3w) &= w(w+8)(1.5) \\ (w-1)(3w) &= (w^2+8w)(1.5) \\ 2w(w-1) &= w(w+8) \\ 2(w-1) &= w+8 \text{ (since } w \neq 0) \\ 2w-2 &= w+8 \\ w &= 10 \end{aligned}$$

Thus, the original dimensions of the rectangle are 10 cm by 18 cm which gives an area of $\boxed{180\,\text{cm}^2}$.

17. Evaluate: $236 \times 542 + 458 \times 764 + 542 \times 764 + 236 \times 458$.
Answer: 1000000
Let $a = 236, b = 542, c = 458, d = 764$
We have now $ab + cd + bd + ac$ which can be factored as $(a + d)(b + c)$. Plugging in the respective values,

$$(a + b)(c + d) = (236 + 764)(542 + 458) = 1000^2 = \boxed{1000000}$$

18. TRUE or FALSE: If n is a real number, then n^2 is positive.
Answer: FALSE
When $n = 0$, $n^2 = 0$ which is neither positive nor negative.

19. If n kilos of rice costs p pesos, how much will x kilos of rice cost?
Answer: $\mathbf{\frac{px}{k}}$ pesos
Rate $= \dfrac{p}{k}$ pesos per kilo. thus, x kilos is worth $\boxed{\dfrac{px}{k}}$ pesos.

20. If the letters of the word MATHEMATICS are repeatedly and consecutively written, what is the 2016th letter?
Answer: T
There are 11 letters on the word so every 11 letters, it repeats the sequence. Thus the remainder when 2016 is divided by 11 gives the 2016th term. $2016 \equiv 3 mod 11$. The third letter of MATHEMATICS is $\boxed{T}$.

21. Simplify: $\dfrac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x^3-2x}$.
Answer: $\mathbf{\dfrac{1}{x}}$

$$\begin{aligned}\frac{\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right)}{x^3-2x} &= \frac{x^2-2}{x(x^2-2)} \\ &= \boxed{\frac{1}{x}}\end{aligned}$$

22. If x is three times as far from -5 as it is from 15, what are the possible values of x?
Answer: $\mathbf{x = \{25, 10, 20\}}$

The equation is $|x+5| = 3|x-15|$.

Case 1: If $x < -5$,

$$\begin{aligned}-x-5 &= 3(-x+15) \\ -x-5 &= -3x+45 \\ 2x &= 50 \\ x &= 25\end{aligned}$$

But $x < -5$, so the solution is extraneous.

Case 2: If $-5 \le x < 15$,

$$\begin{aligned}x+5 &= 3(-x+15) \\ x+5 &= -3x+45 \\ 4x &= 40 \\ x &= \boxed{10}\end{aligned}$$

Case 3: If $x \geq 15$,

$$\begin{aligned} x+5 &= 3(x-15) \\ x+5 &= 3x-45 \\ 2x &= 50 \\ x &= \boxed{25} \end{aligned}$$

23. If three children eat 4 kilos of rice in 5 days, how long will 12 children eat 48 kilos of rice?
Answer: 15 days

Since the number of children is directly proportional to the number of kilos they eat and inversely proportional to the number of days they take.

$$C = k\frac{K}{D} \rightarrow k = \frac{CD}{K}$$

where k is the constant of proportion.

$$\begin{aligned} k &= \frac{3 \cdot 5}{4} = \frac{12 * D}{48} \\ D &= \boxed{15\,\text{days}} \end{aligned}$$

24. A conical tank full of water is emptied into an empty cylindrical tank of the same height. If the base radius of the cylinder is twice that of the cone, what fraction of the cylinder will be filled with water?
Answer: $8\frac{1}{3}$ %

Let the common height be h and the radius of the cone is r so the radius of the cylinder is $2r$.
The volume of the cone therefore is $\frac{1}{3}\pi r^2 h$ while the cylinder has a volume $\pi(2r)^2 h = 4\pi r^2 h$. The required answer is the ratio of the volumes of the volume of the cone to the volume of the cylinder.

$$\frac{V_{cone}}{V_{cylinder}} = \frac{1}{12} = \boxed{8\frac{1}{3}\%}$$

25. Find the minimum integer n for which $\dfrac{18}{n+1}$ is an integer.
Answer: -19

n is minimized when $n+1 = -18$. Hence, $n = \boxed{-19}$.

26. The sum of the square roots of two positive integers is 5. If the two integers differ by 5, what are the integers?
Answer: 9 and 4

If a is the square root of the larger integer, the square root of the smaller integer is $5-a$.
Expressing the second condition,

$$\begin{aligned} a^2-(5-a)^2 &= 5 \\ 10a-25 &= 5 \\ 10a &= 30 \\ a &= 3 \end{aligned}$$

So the square roots of the two integers are 3 and 2. Thus the integers are $\boxed{9,4}$

27. A worm crawls 7.5 inches in 80 seconds. What is its speed in feet per hour?
Answer: 28.125

$$\begin{aligned} \text{speed} &= \frac{7.5\,\text{in}}{80\,\text{s}} \\ &= \frac{7.5\,\text{in}}{80\,\text{s}} \cdot \frac{1\,\text{ft}}{12\,\text{in}} \cdot \frac{3600\,\text{s}}{1\,\text{hr}} \\ &= \boxed{28.125\,\text{ft per hr}} \end{aligned}$$

28. By how much is $(3x-5)(x+2)$ greater than $(x+4)(2x-1)$?
Answer: $\mathbf{x^2-6x-6}$

$$\begin{aligned} (3x-5)(x+2)-(x+4)(2x-1) &= (3x^2+x-10)-(2x^2+7x-4) \\ &= 3x^2+x-10-2x^2-7x+4 \\ &= \boxed{x^2-6x-6} \end{aligned}$$

29. A game consists of drawing a number from $1-20$. A player wins if the number drawn is either a prime number or a perfect square. What is the probability of winning in this game?
Answer: $\mathbf{\frac{3}{5}}$

The prime numbers in that range are: 2,3,5,7,11,13,17, and 19 while the perfect squares are: 1,4,9, and 16. Which means there are 12 numbers that give a player the win. Thus, the probability of winning the game is $\frac{12}{20}=\boxed{\frac{3}{5}}$.

30. The number of boys in a class is equal to the number of girls. Nine boys are absent today, and this leaves twice as many girls as boys in the classroom. How many students belong to the class?
Answer: 36

Let n be the number of girls and boys in the class. So,

$$n = 2(n-9) \to n = 18$$

So the total number of students in the class is $2n$ which is $\boxed{36}$.

31. A square region is removed from a rectangular region. Which of the following can be true?

(a) The perimeter is decreased.
(b) The perimeter is not changed.
(c) The perimeter is increased.

Answer: B and C

When the square region has a common vertex to the rectangle, the perimeter won't change. Otherwise, the perimeter increases.

32. A bag contains 5 black, 5 red, 6 blue, 7 white, 8 yellow, and 10 orange beads. At least how many beads must be drawn from the bag to ensure that at least 3 beads of the same color are chosen?
Answer: 13

The worst case scenario occurs when you have already picked 2 beads from each color and surely after picking the next bead, it will have already three beads of the same color. Thus there should be $2(6) + 1 = \boxed{13}$ beads that will be drawn.

33. How many positive numbers less than 1000 are divisible by 6 but not by 5?
Answer: 133

Counting the number of integers less than 1000 that are divisible by 6:

$$1 < 6n < 1000 \to 1 \leq n \leq 166$$

So there are $166 - 1 + 1 = 166$ numbers that are divisible by 6.

To remove the numbers divisible by 5 out of the 166 multiples of 6, the multiples of 30 on that set should be removed.

$$1 < 30n < 1000 \to 1 \leq n \leq 33$$

Which is 33.

Therefore the answer is $166 - 33 = \boxed{133}$

34. A $4\,\text{cm} \times 5\,\text{cm} \times 7\,\text{cm}$ rectangular prism is painted on all faces. If the prism is cut into $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$ how many cubes do not have paint on any of its sides?
Answer: 30

It is simplfy $(4-2)(5-2)(7-2) = \boxed{30}$ cubes.

35. Andrew is 5 years old and Charlie is 26. In how many years will Charlie be $2\frac{1}{2}$ times as old as Andrew?
Answer: 9 years

$$\begin{aligned} 26+n &= \frac{5}{2}(5+n) \\ 52+2n &= 25+5n \\ n &= \boxed{9} \end{aligned}$$

36. A car is driving along a highway at 55 kph. The driver notices a bus, $\frac{1}{2}$ km behind. The bus passes the car one minute later. What was the speed of the bus?
Answer: 85 kph

Using the formula rate $\cdot$ time $=$ distance,and letting r be the speed of the bus,

$$\begin{aligned} r\left(\frac{1}{60}\right) - 55\left(\frac{1}{60}\right) &= \frac{1}{2} \\ r-55 &= 30 \\ r &= 85 \end{aligned}$$

So the speed of the bus is $\boxed{85\,\text{kph}}$.

37. With what polynomial must $8x^5 - 10x^3 + 2x + 5$ be divided to get a quotient of $4x^2 - 3$ and a remainder of $5 - x$?
Answer: $\mathbf{2x^3 - x}$

$$\begin{aligned} 8x^5 - 10x^3 + 2x + 5 &= P(4x^2-3) + 5 - x \\ P &= \frac{8x^5 - 10x^3 + 2x + 5}{4x^2-3} \end{aligned}$$

$$\begin{array}{r l}
 & 2x^3 \quad - x \\
4x^2 - 3 \big) & 8x^5 - 10x^3 + 3x \\
 & -8x^5 + 6x^3 \\
 & \quad -4x^3 + 3x \\
 & \quad 4x^3 - 3x \\
 & \qquad 0
\end{array}$$

Thus, $P = \boxed{2x^3 - x}$

38. The volume of a sphere is equal to its surface area. What is the diameter of the sphere?
Answer: 6

$$\begin{aligned}
\frac{4}{3}\pi r^3 &= 4\pi r^2 \\
r &= 3
\end{aligned}$$

So the radius is 3 which means the diameter is $\boxed{6}$.

39. If $\overline{3A54B10}$ is divisible by 330, what are the values of A and B?
Answer: $\mathbf{A = 4, B = 1}$

Since $330 = 3 * 10 * 11$, $\overline{3A54B10}$ should be divisible by 3, and 11 to make it divisible by 330 (10 is not needed already since the last digit is 0).
Divisible by 3:

$$A + B + 22 = 27, 36 \rightarrow A + B = 5, 14$$

Divisible by 11:

$$3 - A + 5 - 4 + B - 1 + 0 = -A + B + 3 = 0, 11 \rightarrow -A + B = -3, 8$$

It forms 4 systems of of equations but solving the 4 systems of 2 equations each, there will only be one pair that is acceptable which is $\boxed{A = 4, B = 1}$.

40. Find the solution set: $|5 - 2x| < 19$.
Answer: $\mathbf{-7 < x < 12}$

$$\begin{array}{rcl}
 & |5 - 2x| & < 19 \\
-19 < & 5 - 2x & < 19 \\
-24 < & -2x & < 14 \\
12 > & x & > -7 \\
 & \boxed{-7 < x < 12} &
\end{array}$$

41. If $\overline{47A2969}$ is the square of $3(721 + A)$, find the digit A.
Answer: 8

The square of $3(721+A)$ is $9(721+A)^2$. Which means $\overline{47A2969}$ should be divisible by 9.

$$4 + 7 + A + 2 + 9 + 6 + 9 = A + 37 = 45 \rightarrow A = 8$$

It only has one possible value which is $\boxed{A = 8}$. Verifying it,

$$[3(721 + 8)]^2 = 4782969$$

42. Consider the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, \ldots$. How many numbers in the sequence are needed so that the sum of the reciprocals is 100?
Answer: 5050

Getting the sum of reciprocals, it simplifies to

$$\begin{aligned} \underbrace{1 + 1 + 1 + ... + 1}_{\text{n times}} &= 100 \\ n &= 100 \end{aligned}$$

So we need one hundred 1's which means we need the sequence to be untill the $100th$ 100. Which, in total, has $\frac{100(101)}{2} = \boxed{5050}$ numbers.

43. If n is a positive odd integer, which of the following is a perfect square: $2^{n^2}, 25(3^{n+2}), 7^{n(n+1)}$?
Answer: $\mathbf{7^{n(n+1)}}$

The exponent with an even exponent will be a perfect square. Since n is odd, n^2, $n+3$ will also be odd. So we only need to prove that $n(n + 1)$ is even.
Let $n = 2k + 1$

$$n(n + 1) = (2k + 1)(2k + 2) \rightarrow n(n + 1) = 2(2k + 1)(k + 1)$$

Since there is a actor of 2, $n(n + 1)$ is surely an even number. Thus the only perfect square in that set is $\boxed{7^{n(n+1)}}$.

44. The sum of the first 50 odd integers is 2500. What is the sum of the next 50 odd integers?
Answer: 7500

The sum of the first n odd integers is actually n^2. So to get the required sum, we need to get the sum of the first 100 odd integers, which is 10000 and then subtracting it to 2500 which gives $\boxed{7500}$.

45. Felix has an average of 90 in five tests, each test is with 100 points. What is the lowest possible score Felix could have gotten in a test?
Answer: 50

The sum of his scores in his 5 tests is $5(90) = 450$. To minimize his score in one of the tests, we need to maximize the other 4 tests. Since the maximum value of the score in one test is 100, the 4 tests have a maximum total of 400 points. Thus the minimum possible score in one of Felix's tests is $\boxed{50}$.

46. In square $ABCD$, P is the midpoint of AB and Q is the midpoint of BC. What percent of the area of $ABCD$ is the area of $\triangle PQD$?
Answer: 37.5%

Denote $[XYZ]$ be the area of a polygon XYZ. Now, note that

$$[PQD] = [ABCD] - [PBQ] - [PAD] - [QCD]$$

Getting the areas needed in the RHS by letting one side of the square be n,

$$\begin{aligned} [ABCD] &= n^2 \\ [PBQ] = \frac{1}{2}\left(\frac{n}{2}\right)\left(\frac{n}{2}\right) &= \frac{n^2}{8} \\ [PAD] = \frac{1}{2}n\left(\frac{n}{2}\right) &= \frac{n^2}{4} \\ [QCD] = \frac{1}{2}n\left(\frac{n}{2}\right) &= \frac{n^2}{4} \end{aligned}$$

Hence, $[ABC] = \frac{3}{8}n^2$, i.e $\boxed{37.5\%}$ of $[ABCD]$.

47. If $1 < x < \frac{9}{8}$, which is bigger, $\sqrt[3]{3x}$ or $\sqrt{2x}$?
Answer: $\sqrt[3]{3x}$

$$\begin{array}{rcl} \sqrt[3]{3x} & ??? & \sqrt{2x} \\ (3x)^2 & ??? & (2x)^3 \\ 9x^2 & ??? & 8x^3 \\ \frac{9}{8} & ??? & x \end{array}$$

But $1 < x < \frac{9}{8}$. So

$$\frac{9}{8} \, ??? \, x \to \frac{9}{8} > x$$

Thus LHS is greater than RHS. Since $\boxed{\sqrt[3]{3x}}$ is originally at the LHS, $\sqrt[3]{3x} > \sqrt{2x}$ in the interval $1 < x < \dfrac{9}{8}$.

48. Find the least positive integer that leaves remainders of 1, 2, and 3 when divided by 3, 5, and 7, repsectively.
Answer: 52

Let $n \equiv 1 \bmod 3 \equiv 2 \bmod 5 \equiv 3 \bmod 7$ Since $n \equiv 1 \bmod 3$, $n = 3a+1$ for any positive integer a.

$$\begin{aligned} 3a+1 &\equiv 2 \bmod 5 \\ 3a &\equiv 1 \bmod 5 \\ 6a &\equiv 2 \bmod 5 \end{aligned}$$

Since $5a \equiv 0 \bmod 5$, $(6a - 5a) \equiv (2 - 0) \bmod 5$ or $a \equiv 2 \bmod 5$.
So, $a = 5b + 2$ or $n = 3(5b + 2) + 1 = 15b + 7$.
Finally,

$$\begin{aligned} 15b+7 &\equiv 3 \bmod 7 \\ 15b &\equiv -4 \bmod 7 \\ b &\equiv 3 \bmod 7 \end{aligned}$$

Therefore, $b = 7c + 3$, i.e, $n = 15(7c + 3) + 7 = 105c + 52$. Hence the smallest positive integer n is when $c = 0$ which is $\boxed{52}$.

49. Find all points x on the real number line such that the sum of the distances from x to 4 and from x to -4 is 12.
Answer: $\mathbf{6, -6}$

The equation needed is $|x - 4| + |x + 4| = 12$.

(a) When $x < -4$

$$\begin{aligned} -x+4-x-4 &= 12 \\ -2x &= 12 \\ x &= -6 \end{aligned}$$

(b) When $-4 < x \leq 4$

$$\begin{aligned} -x+4+x+4 &= 12 \\ 8 &\neq 12 \end{aligned}$$

Gives no solution.

(c) When $x \geq 4$

$$\begin{aligned} x - 4 + x + 4 &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Thus, the points are located at $\boxed{6, \text{ and } -6}$.

50. If $11n$ leaves a remainder of 6 when divided by 7, what is the remainder when $5n$ is divided by 7?
Answer: 4

$$\begin{aligned} 11n &\equiv 6 \bmod 7 \\ 4n &\equiv 6 \bmod 7 \\ 2n &\equiv 3 \bmod 7 \\ 2n &\equiv -4 \bmod 7 \\ n &\equiv -2 \bmod 7 \\ n &\equiv 5 \bmod 7 \\ 5n &\equiv 25 \equiv \boxed{4} \bmod 7 \end{aligned}$$

2.2 Grade 8

1. Simplify $(a-b)^2(a+b)^2+2a^2b^2$.
   Answer: $\mathbf{a^4+b^4}$

$$\begin{aligned}(a-b)^2(a+b)^2+2a^2b^2 &= [(a+b)(a-b)]^2+2a^2b^2\\ &= \left(a^2-b^2\right)^2+2a^2b^2\\ &= (a^4-2a^2b^2+b^4)+2a^2b^2\\ &= \boxed{a^4+b^4}\end{aligned}$$

2. Simplify $\left(\dfrac{125x^4y^3}{27x^{-2}y^6}\right)^{\frac{1}{3}}$.
   Answer: $\mathbf{\dfrac{5x^2}{3y}}$

$$\begin{aligned}\left(\frac{125x^4y^3}{27x^{-2}y^6}\right)^{\frac{1}{3}} &= \left(\frac{5^3x^6}{3^3y^3}\right)^{\frac{1}{3}}\\ &= \boxed{\frac{5x^2}{3y}}\end{aligned}$$

3. Solve for x in the equation $x^4-5x^2+4=0$.
   Answer: $\mathbf{\pm 1, \pm 2}$

$$\begin{aligned}x^4-5x^2+4 &= 0\\ (x^2-1)(x^2-4) &= 0\\ (x+1)(x-1)(x+2)(x-2) &= 0\\ x &= \boxed{\pm 1, \pm 2}\end{aligned}$$

4. In the arithmetic sequence $10+10\sqrt{3}, 11+9\sqrt{3}, 12+8\sqrt{3}, \cdots$, what term has no $\sqrt{3}$?
   Answer: 20

$a_n = a_1+(n-1)d \to a_n = \left(10+10\sqrt{3}\right)+(n-1)\left(1-\sqrt{3}\right) = (n+9)+(11-n)\sqrt{3}$
Thus, the term that has no $\sqrt{3}$ is when $n=11$, i.e, $a_{11} = \boxed{20}$

5. If $x + y = 12$ and $xy = 50$, what is $x^2 + y^2$?
Answer: 44

Note that
$$x^2 + y^2 = (x + y)^2 - 2xy$$
Plugging in the values of $x + y$ and xy,
$$x^2 + y^2 = 12^2 - 2(50) = \boxed{44}$$

6. What is the sum of the first ten terms of the geometric sequence 4, 8, 16, $\cdots$,?
Answer: 8188

$$S_n = a\left(\frac{r^n - 1}{r - 1}\right)$$
$$S_{11} = 4\left(\frac{2^{11} - 1}{2 - 1}\right) = \boxed{8188}$$

7. If the product of two consecutive odd integers is 783, what is the sum of their squares?
Answer: 1570

- Solution 1
By letting x and y be the two integers where $x > y$,we have
$x - y = 2$ and $xy = 783$.
Since $x^2 + y^2 = (x - y)^2 + 2xy$,
$x^2 + y^2 = 2^2 + 2(783) = \boxed{1570}$
- Solution 2
Since $783 = 3^3 \cdot 29 = 27 \cdot 29$ and luckily 27 and 29 are consecutive odd integers, we have $27^2 + 29^2 = \boxed{1570}$.

8. If r and s are the roots of the equation $2x^2 - 3x + 4 = 0$, what is $4r^2 + 7rs + 4s^2$?
Answer: 7

Notice that $4r^2 + 7rs + 4s^2 = 4(r + s)^2 - rs$. By Vieta's Identities, $r + s = \frac{3}{2}$ and $rs = 2$. Thus, $4(r + s)^2 - rs = 4\left(\frac{3}{2}\right) - 2 = \boxed{7}$.

9. A long wire is cut into three smaller pieces in the ratio of 7 : 3 : 2. If the shortest piece is 16 cm, what is the area of the largest rectangle that can be created using the longest piece?
Answer: 196 sq. cm

The three smaller pieces can be expressed in the lengths $7k, 3k$, and $2k$, so $2k = 16$ or $k = 8$. The longest piece, therefore, is 56 cm long.
The largest rectangle that can be formed is a square; so its perimeter is 56 cm.
$4s = 56 \to s = 14\text{cm}$. Hence, the area is $\boxed{196\,\text{cm}^2}$.

10. A boat takes $\frac{2}{3}$ as much time to travel downstream as to its return. If the rate of the river's current is 8 kph, what is the rate of the boat in still water?
Answer: 40 kph

Let b be the rate of the boat in still water.

$$\begin{aligned} b - 8 &= \frac{2}{3}(b+8) \\ 3b - 24 &= 2b + 16 \\ b &= \boxed{40\,\text{kph}} \end{aligned}$$

11. How many prime numbers between 40 and 240 ends with 4?
Answer: 0

A number is divisible by 2 and is composite if its last digit is even. The only time that an even number is prime when the number is 2. Since 4 is even, there are $\boxed{0}$ primes that satisfy the condition.

12. Simplify: $x(1 - 6x) - (1 - 2x)(3x - 2)$.
Answer: $\mathbf{-6x + 2}$

$$\begin{aligned} x(1 - 6x) - (1 - 2x)(3x - 2) &= (x - 6x^2) - (-6x^2 + 7x - 2) \\ &= \boxed{-6x + 2} \end{aligned}$$

13. What is the last digit if 7^{2016}?
Amswer:

- Solution 1
Since $\phi(10) = 4$, $7^4 \equiv 1 \bmod 10$. Thus, $7^{2016} = \left(7^4\right)^{504} \equiv 1^{504} = \boxed{1} \bmod 10$

- Solution 2
Getting the last digits of the powers of 7,
$7^1 \to 7, 7^2 \to 9, 7^3 \to 3, 7^4 \to 1$, and repeats the pattern, 7,9,3,1, every 4th power.
Since 2016 is divisible by 4, the fourth term will be the last needed digit which is $\boxed{1}$.

14. If $a = 3$ and $b = 7$, what is $4a^3b + 6a^2b^2 + 4ab^3$?
Answer: 7518

Note that $4a^3b + 6a^2b^2 + 4ab^3 = 2ab(2a^2 + 3ab + 2b^2) = 4ab(a + b)^2 - 2(ab)^2$.
Substitutiong the values, $4(3)(7)(10)^2 - 2(21)^2 = \boxed{7518}$

15. What is the median of the numbers $a + 1, a + 3, a - 2, a + 5,$ and $a - 4$?
Answer: $\mathbf{a + 1}$

Arranging the numbers in increasing order, $a - 4, a - 2, a + 1, a + 3, a + 5$, the median is $\boxed{a + 1}$

16. A rectangle is formed by putting two squares side by side. If each square has perimeter 28 cm, what is the perimeter of the rectangle?
Answer: 42 cm

The side of each square is 7 cm. Referring to its figure, the perimeter will be $\boxed{42\,\text{cm}}$.

17. Solve for x: $4(1 - 3x) - 2x(1 - 3x) + 5(1 - 3x) + 3x(1 - 3x) = 0$.
Answer: $\mathbf{\frac{1}{3}, -9}$

$$\begin{aligned} 4(1 - 3x) - 2x(1 - 3x) + 5(1 - 3x) + 3x(1 - 3x) &= 0 \\ (1 - 3x)(4 - 2x + 5 + 3x) &= 0 \\ (1 - 3x)(x + 9) &= 0 \\ x &= \boxed{\frac{1}{3}, -9} \end{aligned}$$

18. A jacket was worth Php 1200. hoping to gain more profit, the shop owner increased its price by 10 %, but was later forced to reduce it by 15 % since there were no takers. What was the final price of the jacket?
Answer:

The final price is simply $1200(0.9)(0.85) = 1200\left(\frac{9}{10}\right)\left(\frac{17}{20}\right) = \boxed{\text{Php}\,918}$.

19. Let $\{a_n\}$ be an arithmetic sequence. If $a_4 = 27$ and $a_9 = 67$, what is a_1?
Answer: 3

Using the general term for an arithmetic sequence, $a_n = a_1 + (n - 1)d$,the equations will be

$$\begin{aligned} a_4 = a_1 + 3d &= 27 \\ a_9 = a_1 + 8d &= 67 \end{aligned}$$

From the two equations, the first term, $a_1 = \boxed{3}$.

20. Triangle ABC is isosceles with $AB = AC$. Let D be the foot of the altitude from A on BC, and let E be the point on side AC such that DE bisects $\angle ADC$. If $\angle DEC = 67°$, what is $\angle BAC$?
Answer: 34°

Applying properties of isosceles triangles, $\angle BAC = 2\angle DAC$. Also, DE bisects $\angle ADC$ which gives $\angle ADE = \angle EDC = 45°$.
By methods of angle chasing, $\angle ECD = 180° - \angle DEC - \angle EDC = 73°$ and $\angle DAC = 90° - \angle ACD = 17°$. Thus, $\angle BAC = \boxed{34°}$.

21. What is the greatest integer less than or equal to $\left(2+\sqrt{3}\right)^2$?
Answer: 13

$\left(2+\sqrt{3}\right)^2 = 7 + 4\sqrt{3}$. Also, $\sqrt{3} \approx 1.7$. So, $7 + 4(1.7) \approx \boxed{13}$.

22. Each week, a pet owner buys m kilograms of bananas for its monkeys. If each mokey eats n kilograms of bananas each day, how many monkeys does the owner have? Express your answer in terms of m and n.
Answer: $\mathbf{\frac{m}{n}}$

Let k be the number of monkeys. $nk = m \to k = \boxed{\frac{m}{n}}$.

23. If $8.07^3 = 525.557943$, what is 0.807^3?
Answer: 0.525557943

$$\begin{aligned} 0.807^3 &= \left(\frac{8.07}{10}\right)^3 \\ &= \frac{8.07^3}{1000} \\ &= \frac{525.557943}{1000} \\ &= \boxed{0.525557943} \end{aligned}$$

24. Solve for x: $3 < |1 + 2x|$.
Answer: $(-\infty, -\mathbf{2}) \cup (\mathbf{1}, \infty)$ or $\mathbf{x < -2, x > 1}$

$$\begin{aligned} |1+2x| &> 3 \\ 1+2x &\in (-\infty,-3)\cup(3,\infty) \\ 2x &\in (-\infty,-4)\cup(2,\infty) \\ x &\in \boxed{(-\infty,-2)\cup(1,\infty)} \\ \text{or} & \quad \boxed{x<-2, x>1} \end{aligned}$$

25. What is the value of $\frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99}$?
Answer: $\mathbf{\frac{5}{11}}$

Notice that

$$\begin{aligned} \frac{1}{3}+\frac{1}{15}+\frac{1}{35}+\frac{1}{63}+\frac{1}{99} &= \frac{1}{2}\left[\left(1-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{7}\right)+\left(\frac{1}{7}-\frac{1}{9}\right)+\left(\frac{1}{9}-\frac{1}{11}\right)\right] \\ &= \frac{1}{2}\left(1-\frac{1}{11}\right) \\ &= \boxed{\frac{5}{11}} \end{aligned}$$

26. Liza ran 200 meters in only 45 seconds. What was Lisa's speed in kilometers per hour?
Answer: 16 km/h

$$v = \frac{200\,\text{m}}{45\,\text{s}}\cdot\frac{1\,\text{km}}{1000\,\text{m}}\cdot\frac{3600\,\text{s}}{1\,\text{hr}} = \boxed{16\,\text{km/h}}$$

27. Suppose $f(x)$ is a linear function such that $f\left(-\frac{1}{2}\right)=-7$ and $f(1)=-3$. What is $f(-3)$?
Answer: $\mathbf{-\frac{41}{3}}$

The function can be expressed as $f(x)=mx+b$.

$$\begin{aligned} f\left(-\frac{1}{2}\right)=-\frac{1}{2}m+b &= -7 \\ f(1)=m+b &= -3 \end{aligned}$$

From the two equations, $m=\frac{8}{3}, b=-\frac{17}{3}$. So $f(x)=\frac{8}{3}x-\frac{17}{3}$.

Which means $f(-3)=\boxed{-\frac{41}{3}}$.

28. Anna has four cardboard squares, each of which has side of length 6 cm. She decides to form a trapezoid by putting three squares side-by-side, cutting one square along a diagonal, discarding one-half and putting the other half at one end of the squares. What is the area of the trapezoid?
Answer: 126 sq. cm

Simply add the area of the three squares and half of the third square.

29. What is the perimeter of the trapezoid in # 28?
Answer: $6\sqrt{2} + 42$ cm

$$P = 3(6) + 6 + 3(6) + 6\sqrt{2} = \boxed{6\sqrt{2} + 42\,\text{cm}}$$

30. The number 9979 is a four-digit number the sum of whose digits is equal to 34. How many such numbers exist?
Answer: 4

(a) Solution 1
Let a, b, c, d be the digits of the four-digit number.

$$\begin{aligned} a + b + c + d &= 34 \\ (9 - a') + (9 - b') + (9 - c') + (9 - d') &= 34 \\ a' + b' + c' + d' &= 2 \end{aligned}$$

Thus, the desired answer is $\binom{5}{2} - \binom{4}{2} = \boxed{4}$

(b) Solution 2
The only 4 digit numbers that has a digit sum of 34 only consists of three 9's and a 7. So there are only $\boxed{4}$ distinct arrangements of three 9's and a 7.

31. If $\underbrace{10^{10} + 10^{10} + \cdots + 10^{10}}_{10\,terms} = 10^x$, what is x?
Answer: 11

$$\begin{aligned} \underbrace{10^{10} + 10^{10} + \cdots + 10^{10}}_{10\,terms} &= 10^x \\ 10\left(10^{10}\right) &= 10^x \\ 10^{11} &= 10^x \\ x &= \boxed{11} \end{aligned}$$

32. If the sum of the reciprocals of the roots of the equation $3x^2 + 7x + k$ is $-\frac{7}{3}$, what is k?
Answer: 3

Letting m, n be the roots of the equation.

$$\begin{aligned}
\frac{1}{m} + \frac{1}{n} &= \frac{m+n}{mn} \\
&= \frac{-\frac{7}{3}}{\frac{k}{3}} \\
-\frac{7}{3} &= -\frac{7}{k} \\
-\frac{3}{7} &= -\frac{k}{7} \\
k &= \boxed{3}
\end{aligned}$$

33. If $x = 1 + \sqrt{3}$, find the value of $\frac{2x^2 - 4x + 8}{3x^2 - 6x + 10}$.
Answer: $\frac{3}{4}$

$x = 1 + \sqrt{3} \rightarrow (x-1)^2 = 3$

$$\begin{aligned}
\frac{2x^2 - 4x + 8}{3x^2 - 6x + 10} &= \frac{2(x-1)^2 + 6}{3(x-1)^2 + 7} \\
&= \frac{2(3) + 6}{3(3) + 7} \\
&= \boxed{\frac{3}{4}}
\end{aligned}$$

34. The sum of the angles of a regular polygon is $2160°$. How many sides does it have?
Answer: 14

$$\begin{aligned}
180(n-2) &= 2160 \\
n - 2 &= 12 \\
n &= \boxed{14}
\end{aligned}$$

35. An iron cube of side 16 cm is melted down and is used to make eight smaller iron cubes of the same size. What is the length of the sides of the smaller iron cube?
Answer: 2 cm

The volume of the iron cube is 16^3 cu. m, i.e, 2^{12} cu. m. Dividing it by 8, that is 2^9 cu. m per small cube. Thus, each cube has side length $\boxed{2\,\text{cm}}$.

36. The sum of two numbers is 20. If one of the numbers is three times the other, what is their product?
Answer: 75

The equations are:

$$\begin{aligned} x + y &= 20 \\ x &= 3y \end{aligned}$$

From the two equations, $y = 5, x = 15$. So their product is $\boxed{75}$.

37. Determine how many ordered pairs (x, y) satisfy the system

$$\begin{aligned} x^2 + y^2 &= 25 \\ x - y &= 5. \end{aligned}$$

Answer: 2

Substituting $x = y + 5$ to the first equation,

$$\begin{aligned} (y+5)^2 + y^2 &= 25 \\ 2y^2 + 10y &= 0 \\ y^2 + 5y &= 0 \\ y(y+5) &= 0 \\ y &= 0, -5 \end{aligned}$$

Thus the ordered pairs that satisfy the system are $(5, 0)$ and $(0, -5)$. So there are $\boxed{2}$ ordered pairs.

38. A 400mL flask containing 40% alcohol mixture and a 600mL flask containing a 60% alcohol mixture are put together in a single large flask. How many percent alcohol is the resulting mixture?
Answer: 50%

$$\frac{(400)(0.4)+(600)(0.6)}{400+600}=\frac{500}{1000}=\boxed{50\%}$$

39. For what values of k will the equation $y = x^2 + kx + k$ cross the x-axis twice?
Answer: $\mathbf{k} \in (-\infty, \mathbf{0}) \cup (\mathbf{4}, \infty)$

The discriminant should be strictly greater than 0.

$$k^2 - 4k > 0 \to \boxed{k \in (-\infty, 0) \cup (4, \infty)}$$

40. A certain farmer only raises chickens and pigs. Altogether, the animals have 57 heads and 158 legs. How many chickens does he have?
Answer: 35

Let c and p be the number of chickens and pigs, respectively. We now have

$$\begin{aligned} c + p &= 57 \\ 2c + 4p = 158 \to c + 2p &= 79 \end{aligned}$$

From the two equations, $c = \boxed{35}$.

41. Solve for x: $|x - 4| = |2x + 1|$
Answer: $\mathbf{1}, -\mathbf{5}$

There will be three critical intervals to solve the equation, $\left(-\infty, -\frac{1}{2}\right)$, $\left[-\frac{1}{2}, 4\right)$, $[4, \infty)$.

- When $\left(-\infty, -\frac{1}{2}\right)$

$$\begin{aligned} -x + 4 &= -2x - 1 \\ x &= \boxed{-5} \\ \text{rechecking, } |-5-4| &= |2(-5)+1| \\ 9 &= 9 \end{aligned}$$

- When $\left[-\frac{1}{2}, 4\right)$

$$\begin{aligned} -x + 4 &= 2x + 1 \\ x &= \boxed{1} \\ \text{rechecking, } |1-4| &= |2(1)+1| \\ 3 &= 3 \end{aligned}$$

- When $[4, \infty)$

$$\begin{aligned} x - 4 &= 2x + 1 \\ x &= -5 \end{aligned}$$

42. The sum of 100 numbers is 34287. The teacher writes all the numbers on the board and proceed as follows: he adds 1 to the first number, 2 to the second number, 3 to the third, and so on, and adds 100 to the last. What is the sum of the new set of numbers?
Answer: 39837

Simply add 34287 to $1 + 2 + 3 + \cdots + 100 = 5050$ which results to $\boxed{39837}$.

43. Express the answer in lowest terms $\dfrac{6a-2}{9a} \cdot \dfrac{9a^2}{24a-8}$.
Answer: $\mathbf{\dfrac{a}{4}}$

$$\begin{aligned} \frac{6a-2}{9a} \cdot \frac{9a^2}{24a-8} &= \frac{2(3a-1)}{9a} \cdot \frac{9a^2}{8(3a-1)} \\ &= \boxed{\frac{a}{4}} \end{aligned}$$

44. The sum of the roots of the quadratic function $x^2 - 4x + 3$ is 4. If all the coefficients of the quadratic are increased by 2, what is the sum of the roots of the new function?
Answer: $\mathbf{\dfrac{2}{3}}$

The coefficient of x will be -2 while the coefficient of x^2 will be 3. So the sum of the roots will now be $-\dfrac{-2}{3} = \boxed{\dfrac{2}{3}}$.

45. What is the constant term in the expansion of $\left(2x^2 + \dfrac{1}{2x}\right)^6$?
Answer:

One term of the expansion can be expressed as $\dbinom{6}{2}(2x^2)^a(2x)^{-b}$ or $\dbinom{6}{2} \cdot 2^{a-b}x^{2a-b}$. The constant term is when the exponent of x is 0. So $2a - b = 0$. Also, $a + b = 6$. Thus, $a = 2, b = 4$. Thus the constant term is $\boxed{\dfrac{15}{4}}$.

46. Suppose that a is a positive number such that the roots of $x^2 - ax + 1 = 0$ differ exactly by 1. What is the value of a?
Answer: $\sqrt{3}$

Expressing the square of the difference of the roots, $a^2 - 2 = 1$. Thus $a = \boxed{\sqrt{3}}$.

47. Two positive integers are relatively prime if they have no comon factor other than 1. How many two-digit numbers are relatively prime with 24?
Answer: 6

Euler's Totient function $\phi(n)$ determines the number of positive integers less than n that are relatively prime to n. $\phi(n) = n\left(1 - \frac{1}{p_1}\right)\left(1 - \frac{1}{p_2}\right)...\left(1 - \frac{1}{p_k}\right)$ where $p_1, p_2, \cdots, p_k$ are the prime factors of n.Since $24 = 2^3 \cdot 3$, $\phi(24) = 24\left(1 - \frac{1}{2}\right)\left(1 - \frac{1}{3}\right) = 8$. Also, there are 2 one-digit positive integers that are relatively prime with 24. Thus there are $8 - 2 = \boxed{6}$ two-digit positive integers that are relatively prime with 24.

48. A motorist travelled a distance of 180 km. If he had driven 30km/h faster, he could have travelled the same distance 1 hour less time. How fast did he drive?
Answer: 60 kph
Let r be the rate of the motorist while the time he needs to travel at r kph is t.

Since the time difference after changing the motorist's speed is 1 hour, using $t = \frac{d}{v}$,

$$\begin{aligned}
\frac{180}{r+30} - \frac{180}{r} &= 1 \\
180r - 180(r+30) &= r(r+30) \\
r^2 + 30r - 5400 &= 0 \\
(r-60)(r+90) &= 0 \\
r &= \boxed{60\,\text{kph}}
\end{aligned}$$

49. Solve for x in the inequality $5 < |1 - 2x| \leq 7$.
Answer: $[-\mathbf{3}, -\mathbf{2}) \cup (\mathbf{3}, \mathbf{4}]$

Splitting the inequality into $|1 - 2x| > 5$ and $|1 - 2x| \leq 7$, we have

(a) $|1 - 2x| > 5$

$$\begin{aligned}
|1-2x| &> 5 \\
1 - 2x &\in (-\infty, -5) \cup (5, \infty) \\
-2x &\in (-\infty, -6) \cup (4, \infty) \\
x &\in \underline{(-\infty, -2) \cup (3, \infty)}
\end{aligned}$$

(b) $|1-2x| \leq 7$

$$\begin{aligned} |1-2x| &\leq 7 \\ 1-2x &\in [-7,7] \\ -2x &\in [-8,6] \\ x &\in \underline{[-3,4]} \end{aligned}$$

Getting the intersection of the two solution sets, gives $x \in \boxed{[-3,-2) \cup (3,4]}$

50. Joe draws a line on the board and marks five points on that line. He then marks two points on the board that lies on the same line, obtaining a total of seven points. How many different triangles can be drawn using the seven points as vertices?
Answer: 25

(a) Case 1: When two vertices are on the line with the 5 points given. There are $\binom{5}{2}$ ways to assign 2 vertices from the 5 given points and there are $\binom{2}{1}$ to assign a vertex from the line with 2 points. Which has a total of $\binom{5}{2}\binom{2}{1} = (10)(2) = 20$

(b) Case 2: When two vertices are the two points on the line with the two given points. There are 5 ways to assign the third vertex.

Thus, the total number of triangles that can be drawn is $20 + 5 = \boxed{25}$.

2.3 Grade 9

1. Solve for x in $\sqrt{2-x}-4=0$.
Answer: −14

$$\begin{aligned} \sqrt{2-x}-4 &= 0 \\ \sqrt{2-x} &= 4 \\ 2-x &= 16 \\ x &= \boxed{-14} \end{aligned}$$

2. Find $\sin\theta$ if $\tan\theta = \frac{3}{4}$ and $\cos\theta < 0$.
Answer: $-\frac{\mathbf{3}}{\mathbf{5}}$

$$\sin\theta = -\frac{3}{\sqrt{3^2+4^2}} = \boxed{-\frac{3}{5}}$$

3. What is the value of $\tan\left(-\frac{17}{6}\pi\right)$?
Answer: $\frac{\sqrt{\mathbf{3}}}{\mathbf{3}}$

$$\tan\left(-\frac{17}{6}\pi\right) = \tan\left(\frac{\pi}{6}\right) = \boxed{\frac{\sqrt{3}}{3}}$$

4. Solve for x in $\frac{5}{x+2} - \frac{2}{x+1} = \frac{1}{2}$.
Answer: 0, 3

$$\begin{aligned} \frac{5}{x+2} - \frac{2}{x+1} &= \frac{1}{2} \\ 10(x+1)-4(x+2) &= (x+2)(x+1) \\ 6x+2 &= x^2+3x+2 \\ x^2-3x &= 0 \\ x(x-3) &= 0 \\ x &= \boxed{0,3} \end{aligned}$$

5. Solve for x in $x^4-4x^2-5=0$.
Answer: $\pm\mathbf{i}, \pm\sqrt{5}$

$$\begin{aligned} x^4 - 4x^2 - 5 &= 0 \\ (x^2-5)(x^2+1) &= 0 \\ x &= \boxed{\pm i, \pm\sqrt{5}} \end{aligned}$$

*Note: $i = \sqrt{-1}$

6. Find the sum of the first 20 odd numbers.
   Answer: 400

   The sum of the first n odd numbers is n^2. Thus, $\boxed{400}$.

   Reference:

$$\begin{aligned} \sum_{k=1}^{n}(2k-1) &= 2\sum_{k=1}^{n} k - \sum_{k=1}^{n} 1 \\ &= 2\left(\frac{n(n+1)}{2}\right) - n \\ &= n(n+1) - n \\ \sum_{k=1}^{n}(2k-1) &= \underline{n^2} \end{aligned}$$

7. Solve for x in $x^2 + 12 < 7x$.
   Answer: $\mathbf{x \in (3, 4)}$

$$\begin{aligned} x^2 + 12 &< 7x \\ x^2 - 7x + 12 &< 0 \\ (x-4)(x-3) &< 0 \\ x &\in \boxed{(3,4)} \end{aligned}$$

8. Find all possible values of x if one of the interior angles of a square is $(2x^2 - 8x)^\circ$.
   Answer: $\mathbf{-5, 9}$

$$\begin{aligned} 2x^2 - 8x &= 90 \\ x^2 - 4x &= 45 \\ x^2 - 4x - 45 &= 0 \\ (x-9)(x+5) &= 0 \\ x &= \boxed{-5, 9} \end{aligned}$$

9. What is the sum of the interior angles of a pentagon?
Answer: 540°

$$180(5-2) = \boxed{540^\circ}$$

10. The supplement of an angle is four times its complement. Find the angle.
Answer: 60°

$$\begin{aligned} 180 - x &= 4(90 - x) \\ 180 - x &= 360 - 4x \\ 3x &= 180 \\ x &= \boxed{60^\circ} \end{aligned}$$

11. What is the shortest side of $\triangle PQR$ if $\angle P = 57^\circ$ and $\angle Q = 65^\circ$?(Revised problem)
Answer: QR

The third angle measures $180^\circ - 57^\circ - 65^\circ = 58^\circ$. The smallest angle is $\angle P$ which means the shortest side is $\boxed{QR}$.

12. Rationalize the denominator and simplify: $\dfrac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}}$.
Answer: $\sqrt{3} - 1$

$$\begin{aligned} \frac{2\sqrt{2}}{\sqrt{6}+\sqrt{2}} &= \frac{2\sqrt{2}}{\sqrt{2}\left(\sqrt{3}+1\right)} \\ &= \frac{2}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} \\ &= \frac{2\left(\sqrt{3}-1\right)}{3-1} \\ &= \boxed{\sqrt{3}-1} \end{aligned}$$

13. Find the 25th term of the arithmetic sequence whose first 3 terms are 4,7, and 10.
Answer: 76

The common difference is 3 and the first term is 4. Thus, the 25th term is $4 + (25 - 1)(3) = \boxed{76}$

14. Solve for x in $\sqrt{2x+1} - \sqrt{x} = 1$.
Answer: 0, 4

$$\begin{aligned}
\sqrt{2x+1} - \sqrt{x} &= 1 \\
\sqrt{2x+1} &= 1 + \sqrt{x} \\
2x+1 &= 1 + x + 2\sqrt{x} \\
x &= 2\sqrt{x} \\
x^2 &= 4x \\
x^2 - 4x &= 0 \\
x(x-4) &= 0 \\
x &= \boxed{0, 4}
\end{aligned}$$

15. For what value/s of x is $x^2 - 8x + 18 \geq 0$?
Answer: $\mathbf{x} \in \Re$

$$\begin{aligned}
x^2 - 8x + 18 &\geq 0 \\
(x-4)^2 + 2 &> 0, \text{ for all } \boxed{x \in \Re}
\end{aligned}$$

16. In $\triangle ABC$, $\angle B = 90°$ and $\sin A = \frac{3}{4}$. Determine the value of $\tan C$.
Answer: $\mathbf{\frac{3\sqrt{7}}{7}}$

$$\tan C = \frac{3}{\sqrt{4^2 - 3^2}} = \boxed{\frac{3\sqrt{7}}{7}}$$

17. The angle bisector at vertex B of $\triangle ABC$ meets AC at D. If $BC = 6, AC = 40$ and $CD = 15$, find AB.
Answer: 10

It shows that $\triangle ADB \sim \triangle CDB$. So,

$$\begin{aligned}
\frac{AD}{CD} &= \frac{AB}{CB} \\
\frac{25}{15} &= \frac{AB}{6} \\
AB &= \boxed{10}
\end{aligned}$$

18. Find the equation of the line parallel to $4x + 3y = 12$ and passing through $(-12, 4)$.
Answer: $\mathbf{4x + 3y + 36 = 0}$

The line is in the form $4x + 3y = k$. Since it passes through $(-12, 4)$, $4(-12) + 3(4) = k \to k = -36$. Thus, $\boxed{4x + 3y + 36 = 0}$.

19. Express in terms of sines and cosines of θ and simplify: $\cot\theta\sec^2\theta$.
Answer: $\dfrac{\mathbf{1}}{\sin\theta\cos\theta}$

$$\begin{aligned}\cot\theta\sec^2\theta &= \frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\cos^2\theta}\\ &= \boxed{\frac{1}{\sin\theta\cos\theta}}\end{aligned}$$

20. Find the common ratio of a geometric sequence whose first term is -2 and the 7th term is -1458.
Answer: $\mathbf{\pm 3}$

$a = -2, ar^6 = -1458$. Dividing the two equations, $r^6 = 729 \to r = \boxed{\pm 3}$.

21. Solve for x in $4\sqrt{3x+1} = 4x + 3$.
Answer: $\mathbf{\frac{7}{4}, -\frac{1}{4}}$

$$\begin{aligned}4\sqrt{3x+1} &= 4x+3\\ 16(3x+1) &= 16x^2+24x+9\\ 16x^2-24x-7 &= 0\\ (4x-7)(4x+1) &=\\ x &= \boxed{\frac{7}{4}, -\frac{1}{4}}\end{aligned}$$

22. Find the 7th term of the geometric sequence: 8, 12, 18, $\cdots$.
Answer: $\mathbf{\frac{729}{8}}$

$$a_7 = 8\cdot\left(\frac{3}{2}\right)^6 = \boxed{\frac{729}{8}}$$

23. Triangle ABC is a right triangle with $C = 90°$. If $A = 60°$ and $a = 50$, find b.
Answer: $\mathbf{\frac{50\sqrt{3}}{3}}$

$$b = \frac{50}{\tan 60°} = \frac{50}{\sqrt{3}} = \boxed{\frac{50\sqrt{3}}{3}}$$

24. Two of the exterior angles of a regular polygon have measures $(6x - 30)°$ and $(114 - 10x)°$. How many sides does this regular polygon have?
Answer: 15

$$\begin{aligned} 6x - 30 &= 114 - 10x \\ 16x &= 144 \\ x &= 9 \\ 6(9) - 30 &= 24° \\ \frac{360°}{n} &= 24° \\ n &= \boxed{15} \end{aligned}$$

25. Solve for x in $\frac{3}{2x-1} - \frac{2}{x} = 0$.
Answer: 2

$$\begin{aligned} \frac{3}{2x-1} - \frac{2}{x} &= 0 \\ \frac{3}{2x-1} &= \frac{2}{x} \\ 3x &= 4x - 2 \\ x &= \boxed{2} \end{aligned}$$

26. Find x so that $x - 2, x + 2$ and $x + 4$ are consecutive terms of a geometric seuqnece.
Answer: −6

$$\begin{aligned} r = \frac{x+4}{x+2} &= \frac{x+2}{x-2} \\ (x+4)(x-2) &= (x+2)^2 \\ x^2 + 2x - 8 &= x^2 + 4x + 4 \\ 2x &= -12 \\ x &= \boxed{-6} \end{aligned}$$

27. What is the smallest positive angle which is co-terminal to $-1125°$?
Answer: 315°

$$-1125 = 315 - 4(360) \to \boxed{315°}$$

28. What is the height of an equilateral triangle whose perimeter is 6 meters?
Answer: $\sqrt{3}$ m

The side measures 2 m. Thus the height is $2\frac{\sqrt{3}}{2} = \boxed{\sqrt{3}\text{ m}}$.

29. By what factor is the volume of a cube increased if each of its sides is tripled?
Answer: 27

A cube of side s has volume s^3. If the sides are tripled, which is, $3s$ in length, the volume will be $27s^3$ which is $\boxed{27}$ times larger than the original.

30. z varies directly as x and varies inversely as the square of y. If $z = \frac{7}{2}$ when $x = 14$ and $y = 6$, find z when $x = 27$ and $y = 9$.
Answer: 3

The equation of variation is in the form

$$z = k\frac{x}{y^2} \to k = \frac{zy^2}{x}$$

where k is the constant of variation. So we have

$$\begin{aligned} k = \frac{\left(\frac{7}{2}\right)6^2}{14} &= \frac{z \cdot 9^2}{27} \\ 9 &= 3z \\ z &= \boxed{3} \end{aligned}$$

31. Express in terms of sines or cosines of θ and simplify: $\frac{\cot^2\theta + 1}{\tan^2\theta + 1}$.

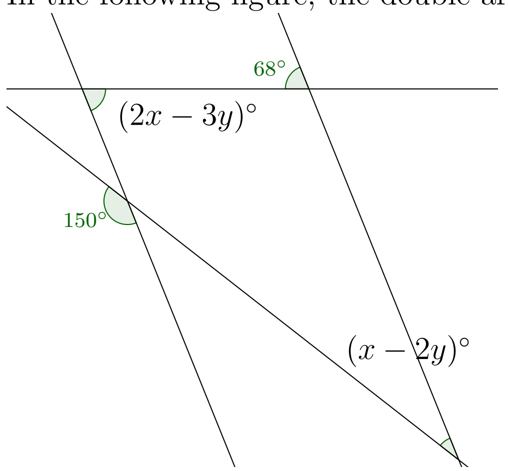
Answer: $\left(\frac{\cos\theta}{\sin\theta}\right)^{\mathbf{2}}$

$$\begin{aligned} \frac{\cot^2\theta + 1}{\tan^2\theta + 1} &= \frac{\csc^2\theta}{\sec^2\theta} \\ &= \frac{\cos^2\theta}{\sin^2\theta} \\ &= \boxed{\left(\frac{\cos\theta}{\sin\theta}\right)^2} \end{aligned}$$

32. Right $\triangle ABC$, with right angle at C, has sides $b = 5$ and $c = 7$. Find $\csc B$.
Answer: $\mathbf{\frac{7}{5}}$

$$\csc B = \frac{c}{b} = \boxed{\frac{7}{5}}$$

33. In the following figure, the double arrows indicate parallel lines. Find x.



Answer: 46

By using properties of parallel lines,

$$\begin{aligned} 2x - 3y &= 68 \\ x - 2y + 150 = 180 \rightarrow x - 2y &= 30 \end{aligned}$$

From the two equations, $x = \boxed{46}, y = 8$.

34. What is the periemter of an equilateral triangle whose area is $75\sqrt{3}$ sq. cm?
Answer: $\mathbf{30\sqrt{3}}$ cm

$$\begin{aligned} s^2\frac{\sqrt{3}}{4} &= 75\sqrt{3} \\ s^2 &= 300 \\ s &= 10\sqrt{3} \\ 3s &= \boxed{30\sqrt{3}\text{ cm}} \end{aligned}$$

35. A person standing 40 ft away from a street light that is 25 ft tall. How tall is he if his shadow is 10 ft long?

Answer: 16 ft

By similar triangles,

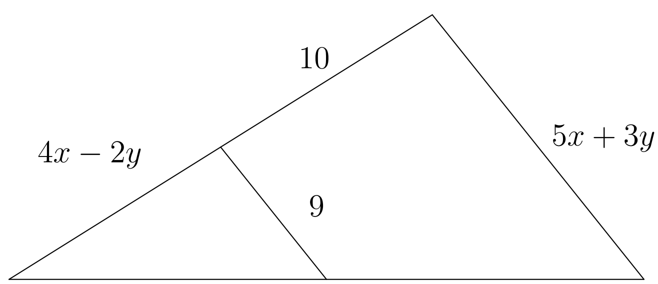
$$\begin{aligned} \frac{x}{40} &= \frac{10}{25} \\ \frac{x}{40} &= \frac{2}{5} \\ x &= \boxed{16\,\text{ft}} \end{aligned}$$

36. What is the maximum value of $f(x) = -2x^2 - 4x + 3$?
Answer: 5

$$\begin{aligned} f(x) &= -2x^2 - 4x + 3 \\ &= -2(x^2 + 2x) + 3 \\ &= -2\left[(x^2 + 2x + 1) - 1\right] + 3 \\ &= -2(x+1)^2 + 2 + 3 \\ &= -2(x+1)^2 + 5 \end{aligned}$$

Maximum is attained when $x = -1$. Thus, $\max f(x) = \boxed{5}$

37. The figure shows a segment joining the midpoints of two sides of a triangle. What is the sum of x and y?



Answer: 4

By properties of midsegments, we have the equations

$$\begin{aligned} 4x - 2y = 10 \rightarrow 2x - y &= 5 \\ 5x + 3y = 2(9) \rightarrow 5x + 3y &= 18 \end{aligned}$$

From the two equations, $x = 3, y = 1$. Thus $x + y = \boxed{4}$.

38. If $x > 1$, is $\frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}}$ positive or negative?
Answer: positive

$$\begin{aligned} \frac{3}{2}x^{\frac{1}{2}} - \frac{3}{2}x^{-\frac{1}{2}} &= \frac{3}{2}\left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) \\ &= \frac{3}{2}x^{-\frac{1}{2}}(x-1) \end{aligned}$$

Since $x^{-\frac{1}{2}}$ is always positive for $x \geq 0$, and $(x-1)$ is positive when $x > 1$ (given), the expression is $\boxed{\text{positive}}$.

39. The diagonals of a rhombus are in the ratio of 1:3. If each side of the rhombus is 10 cm long, find the length of the longer diagonal.
Answer:

Let the diagonals be k and $3k$ in length. By the pythagorean theorem,

$$\begin{aligned} \left(\frac{k}{2}\right)^2 + \left(\frac{3k}{2}\right)^2 &= 10^2 \\ \frac{k^2}{4} + \frac{9k^2}{4} &= 100 \\ 10k^2 &= 400 \\ k &= 20 \\ 3k &= \boxed{60\,\text{cm}} \end{aligned}$$

40. Find a and b so that the zeroes of $ax^2 + bx + 24$ are 3 and 4.
Answer: $\mathbf{a = 7, b = -29}$

If 3 and 4 are factors, then they should satisfy:

$$\begin{aligned} a \cdot 3^2 + b \cdot 3 + 24 = 0 \to 9a + 3b = -24 \to 3a + b &= -8 \\ a \cdot 4^2 + b \cdot 4 + 24 = 0 \to 16a + 4b = -24 \to 4a + b &= -6 \end{aligned}$$

Thus, $\boxed{a = 2, b = -14}$.

41. Find all k so that the graph of $y = -\frac{1}{4}x^2 + kx - 9$ is tangent to the x-axis.
Answer: $\pm\mathbf{3}$

The discriminant should be 0.

$$\begin{aligned} D = k^2 - 4\left(-\frac{1}{4}\right)(-9) &= 0 \\ k^2 - 9 &= 0 \\ k &= \boxed{\pm 3} \end{aligned}$$

42. The diagonals of parallelogram $JKLM$ intersect at P. If $PM = 3x - 2, PK = x + 3$ and $PJ = 4x - 3$, find the length of PL.
Answer: 7

Note that $PM = PK \rightarrow 3x-2 = x+3$. So $x = \frac{5}{2}$. Also, $PJ = PL$. So, $4\left(\frac{5}{2}\right)-3 = 7$. Thus, $PJ = 7$.

43. Suppose that w varies directly as x and the square of y and inversely as the square root of z. If x is increased by 80 %, y is increased by 40 %, and z is increased by 44%, by how many percent will w increase?
Answer: 194%

The equation of variation with the constant of variation k is

$$w = k\frac{xy^2}{\sqrt{z}} \rightarrow k = \frac{w\sqrt{z}}{xy^2}$$

Now, increasing x by 80% results to $1.8x = \frac{9}{5}x$, y by 40% to $1.4y = \frac{7}{5}y$, and z by 44% to $1.44z = \frac{36}{25}z$. Also, w will change by a factor of k, i.e, wk So,

$$\begin{aligned} k = \frac{w\sqrt{z}}{xy^2} &= \frac{wk\sqrt{\frac{36}{25}z}}{\left(\frac{9}{5}x\right)\left(\frac{7}{5}y\right)^2} \\ \frac{w\sqrt{z}}{xy^2} &= \frac{wk\sqrt{z}}{xy^2} \cdot \frac{\frac{6}{5}}{\frac{9}{5} \cdot \frac{49}{25}} \\ 1 &= k \cdot \frac{50}{147} \\ k &= \frac{147}{50} \end{aligned}$$

Thus, w will increase by $\frac{147}{50} - 1 = \boxed{194\%}$

44. Find k so that the minimum value of $f(x) = x^2 + kx + 8$ is equal to the maximum value of $g(x) = 1 + 4x - 2x^2$.
Answer: $\pm\mathbf{2}\sqrt{5}$

Getting the maximum value of $g(x) = 1 + 4x - 2x^2$,

$$\begin{aligned} g(x) &= -2x^2 + 4x + 1 \\ &= -2(x^2 - 2x) + 1 \\ &= -2(x^2 - 2x + 1) + 2 + 1 \\ g(x) &= -2(x+1)^2 + 3 \\ \max g(x) &= 3 \end{aligned}$$

Now,

$$\begin{aligned} f(x) &= x^2 + kx + 8 \\ &= \left(x^2 + kx + \frac{k^2}{4}\right) - \frac{k^2}{4} + 8 \\ f(x) &= \left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 8 \\ \min f(x) = 8 - \frac{k^2}{4} &= 3 \\ \frac{k^2}{4} &= 5 \\ k^2 &= 20 \\ k &= \boxed{\pm 2\sqrt{5}} \end{aligned}$$

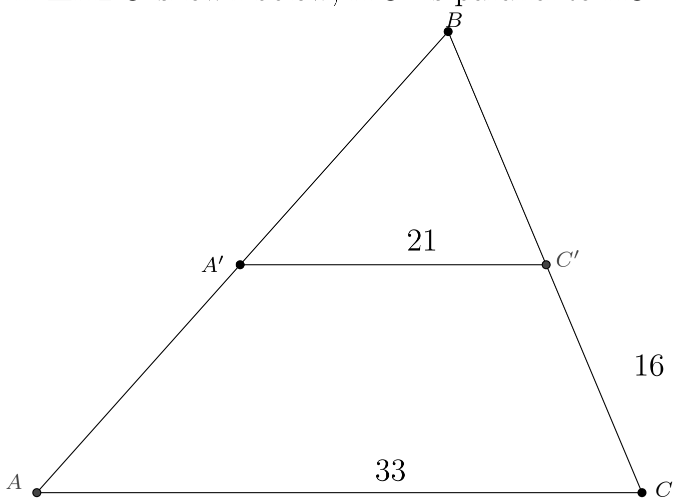
45. The difference of two numbers is 22. Find the numbers so that their product is to be minimum.
Answer: $\mathbf{-11, 11}$

The two numbers are x and $x + 22$. Let $f(x)$ be a function of their product.

$$\begin{aligned} f(x) &= x(x+22) \\ &= x^2 + 22x \\ f(x) &= (x+11)^2 - 121 \end{aligned}$$

$f(x)$ will be minimum if $x = -11$, so $x + 22 = 11$. Thus, the numbers are $\boxed{-11, 11}$.

46. In $\triangle ABC$ shown below, $A'C'$ is parallel to AC. Find the length of BC'.



Answer: 28

By similarity,

$$\begin{aligned}
\triangle ABC &\sim \triangle A'BC' \\
\frac{BC'}{BC} &= \frac{A'C'}{AC} \\
\frac{BC'}{BC'+16} &= \frac{21}{33} \\
\frac{BC'}{BC'+16} &= \frac{7}{11} \\
11BC' &= 7BC' + 112 \\
BC' &= \boxed{28}
\end{aligned}$$

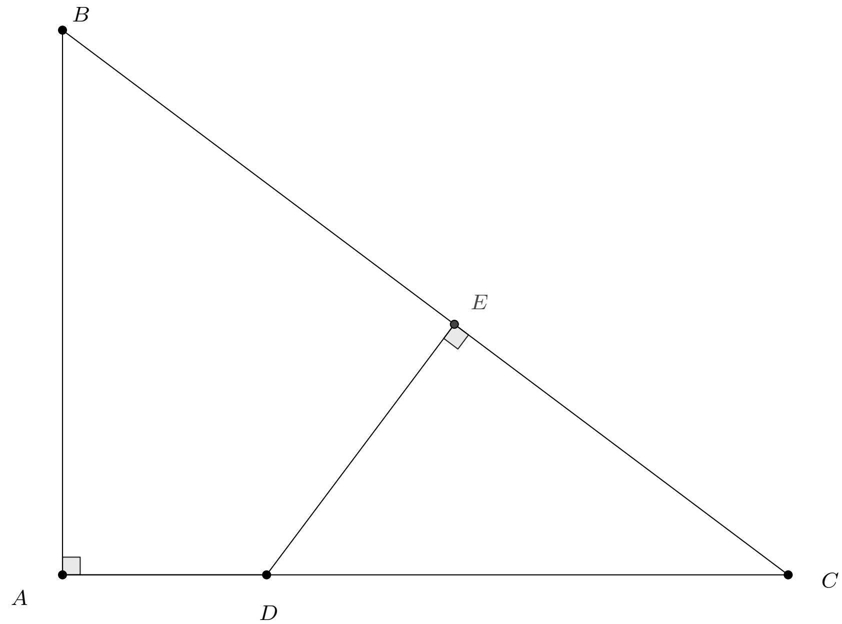
47. Find the length of h, the height drawn to the hypotenuse, of the right $\triangle ABC$ with right angle at C if h divides the hypotenuse into two parts of length 25 (from A) and 9.

Answer: 15

Simply

$$h = \sqrt{ab} = \sqrt{25 \cdot 9} = \boxed{15}$$

48. In the figure below, $\angle BAC$ and $\angle DEC$ are both right angles, $CD = 6, BC = 10$, and the length of AD is one-fourth the length of AC. Find CE.



Answer: $\mathbf{\frac{24}{5}}$

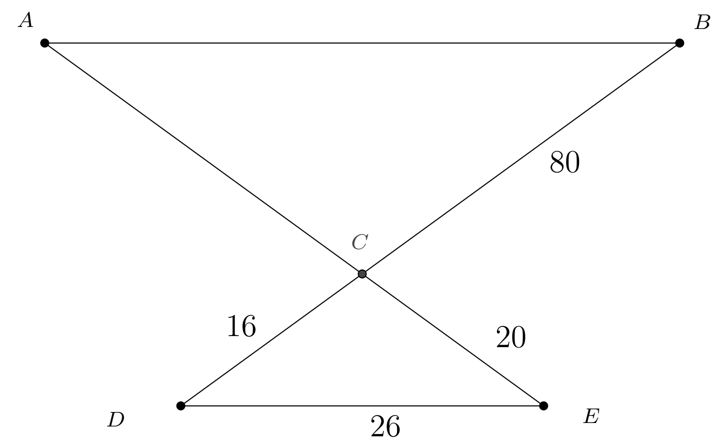
$$\begin{aligned} \triangle BAC &\sim \triangle DEC \\ \frac{BC}{AC} &= \frac{DC}{EC} \\ \frac{10}{8} &= \frac{6}{EC} \\ \frac{5}{4} &= \frac{6}{EC} \\ EC &= \boxed{\frac{24}{5}} \end{aligned}$$

49. The number r varies jointly as s and the square of t. If $r = 6$ when $s = 12$ and $t = \frac{1}{2}$, find r when $s = 18$ and $t = \frac{3}{2}$.
Answer: 1

$$r = k\frac{s}{t^2} \rightarrow k = \frac{rt^2}{s}$$

$$\begin{aligned}\frac{6\left(\frac{1}{2}\right)^2}{12} &= \frac{r\left(\frac{3}{2}\right)^2}{18}\\ \frac{1}{8} &= \frac{r}{8}\\ r &= \boxed{1}\end{aligned}$$

50. Given the figure below with AB parallel to DE. Find the length of AB.



Answer: 130

$$\begin{aligned}\angle ACB &= \angle ECD\\ \angle CAB &= \angle CED\,\text{(PAIC)}\\ \triangle ABC &\sim \triangle EDC\,\text{(AA)}\\ \frac{DC}{BC} &= \frac{ED}{AB}\\ \frac{16}{80} &= \frac{26}{AB}\\ 5 &= \frac{AB}{26}\\ AB &= \boxed{130}\end{aligned}$$

2.4 Grade 10

1. Find the value of $\sqrt{x+\sqrt{14-x}}$ when $x=-2$.
Answer: $\sqrt{2}$

$$\sqrt{-2+\sqrt{14-(-2)}} = \sqrt{-2+\sqrt{16}} = \sqrt{-2+4} = \boxed{\sqrt{2}}$$

2. Find the area of the triangle formed by the coordinate axes and the line $3x+2y=6$.
Answer: $\mathbf{3}\,\text{units}^2$

The intercepts $(0,3),(2,0)$ and and the origin form a right triangle of legs 3 and 2 in length. Thus, having an area of $\boxed{3\,\text{units}^2}$.

3. A sequence is defined by $a_n = 3(a_{n-1}+2)$ for $n \geq 2$, where $a_1 = 1$. What is a_4?
Answer: 105

$$\begin{aligned} a_2 = 3(a_1+2) &= 9 \\ a_3 = 3(a_2+2) &= 33 \\ a_4 = 3(a_3+2) &= \boxed{105} \end{aligned}$$

4. Christa leaves Town A. After traveling 12 km, she reaches Town B at 2:00 P.M. Then she drove at a constant speed and passes Town C, 40 km from Town B, at 2:50 P.M. Find the function $d(t)$ that models the distance (in km) she has traveled from Town A t minutes after 2:00 P.M.
Answer: $\mathbf{d(t) = \frac{14}{25}t + 12}$

Since Christa's speed is constant, $d(t)$ is a linear function, i.e, $d(t) = at+b$. Also, $d(0)=12, d(50)=40$ By solving for a and b, we get $\boxed{d(t) = \frac{14}{25}t+12}$.

5. What is the largest negative integer that satisfies the inequality $|3x+2| > 4$?
Answer: $\mathbf{-3}$

$$\begin{aligned} |3x+2| &> 4 \\ 3x+2 &\in (-\infty,-4)\cup(4,\infty) \\ 3x &\in (-\infty,-6)\cup(2,\infty) \\ x &\in (-\infty,-2)\cup\left(\frac{2}{3},\infty\right) \end{aligned}$$

Thus, the largest negative integer that satisfy the condition is $\boxed{-3}$.

6. A person has two parents, four grandparents, eight grand-parents, and so on. How many ancestors does a person have 10 generations back?
Answer: 2046

The geometric sequence has common ratio of 2 and first term 2. Thus the sum $S_n = 2\left(\frac{2^n - 1}{2 - 1}\right) = 2\left(2^n - 1\right)$. Thus, $S_{10} = \boxed{2046}$.

7. In $\triangle ABC, \angle B$ is twice $\angle A$, and $\angle C$ is three times as large as $\angle B$. Find $\angle C$.
Answer: 120°

From the first 2 conditions, it shows that $\angle C = 6\angle A$. Also, $\angle A + \angle B + \angle C = 180°$. Substitutiong, $A + 2A + 6A = 180 \rightarrow A = 20$. Thus, $\angle C = \boxed{120°}$.

8. If $-3 \leq x \leq 0$, find the minimum value of $f(x) = x^2 + 4x$.
Answer: −2

$$\begin{aligned} f(x) &= x^2 + 4x \\ &= (x^2 + 4x + 4) - 4 \\ f(x) &= (x + 2)^2 - 4 \end{aligned}$$

The minimum value of $f(x)$ is when $x = -2$. Since it is within the interval $-3 \leq x \leq 0$, it is the minimum value. $f(-2) = \boxed{-4}$.

9. The 9th and the 11th term of an arithmetic sequence are 28 and 45, respectively. What is the 12th term?

Answer: $\mathbf{\frac{107}{2}}$

The two equations ,where a is the first term and d is the common difference, are:

$$\begin{aligned} a + 8d &= 28 \\ a + 10d &= 45 \end{aligned}$$

From the two equations, $d = \frac{17}{2}$ and $a = -40$.
So the 12th term is $-40 + (11)\left(\frac{17}{2}\right) = \frac{107}{2}$.

10. Perform the indicated operation, and simplify: $\frac{2}{x} + \frac{3}{x-1} - \frac{4}{x^2 - x}$.
Answer: $\mathbf{\frac{5x - 6}{x^2 - x}}$

$$\begin{aligned}\frac{2}{x}+\frac{3}{x-1}-\frac{4}{x^2-x} &= \frac{2(x-1)+3x-4}{x^2-x}\\ &= \boxed{\frac{5x-6}{x^2-x}}\end{aligned}$$

11. Find the range of the function $f(x) = |2x+1|$.
Answer: $[\mathbf{0}, \infty)$

Since the expression inside the absolute value can be zero and $|a| \geq 0$ for all real x, the range is $f(x) \in \boxed{[0, \infty)}$.

12. Find the value of $[x - (x - x^{-1})^{-1}]^{-1}$ when $x = 2$.
Answer: $\mathbf{\frac{3}{4}}$

$$\begin{aligned}[x - (x - x^{-1})^{-1}]^{-1} &= \left[x - \left(\frac{x^2-1}{x}\right)^{-1}\right]^{-1}\\ &= \left[x - \frac{x}{x^2-1}\right]^{-1}\\ &= \left[\frac{x^3-2x}{x^2-1}\right]^{-1}\\ &= \frac{x^2-1}{x^3-2x}\\ @x = 2 &\rightarrow \frac{2^2-1}{2^3-2(2)}\\ &= \boxed{\frac{3}{4}}\end{aligned}$$

13. Find the equation (in the form $ax + by = c$) of the line through the point $(5, 2)$ that is parallel to the line $4x + 6y + 5 = 0$.
Answer: $\mathbf{2x + 3y = 16}$

The line is in the form $4x + 6y + k = 0$. Since it passes through $(5, 2)$, $4(5) + 6(2) + k = 0 \rightarrow k = -32$. Thus, the equation of the line is $\boxed{2x + 3y = 16}$.

14. In $\triangle ABC, \angle B = 90°, \angle ACB = 28°$, and D is the midpoint of AC. What is $\angle BDC$?
Answer: $\mathbf{124°}$

BD is a median to the hypotenuse which means $BD = DC$. Furthermore, $\angle DBC = \angle ACB = 28°$. Therefore, $\angle BDC = 180 - 2(28) = \boxed{124°}$.

15. Find all solutions of the system: $\begin{cases} 2x+y=1 \\ 3x+4y=14 \end{cases}$.

Asnwer: $\mathbf{x=-2, y=5}$

From equation 1, $y = 1-2x$. Substituting to the second equation,

$$\begin{aligned} 3x+4y &= 14 \\ 3x+4(1-2x) &= 14 \\ -5x+4 &= 14 \\ x &= -2 \end{aligned}$$

Therefore, $\boxed{x=-2, y=5}$.

16. If $f(2x-1)=x$, what is $f(2)$?

Answer: $\mathbf{\frac{3}{2}}$

$$f(2) = f(2x-1) \rightarrow 2 = 2x-1 \rightarrow x = \frac{3}{2}$$

Thus, $f(2) = \boxed{\frac{3}{2}}$.

17. What is the quotient when $3x^5+5x^4-4x^3+7x+3$ is divided by $x+2$?

Answer: $3x^4-x^3-2x^2+4x-1$

$$\begin{array}{r} 3x^4 - x^3 - 2x^2 + 4x - 1 \\ x+2 \overline{\big) \; 3x^5 + 5x^4 - 4x^3 \qquad + 7x + 3} \\ -\,3x^5 - 6x^4 \qquad\qquad\qquad\qquad \\ \hline -x^4 - 4x^3 \qquad\qquad\qquad \\ x^4 + 2x^3 \qquad\qquad\qquad \\ \hline -2x^3 \qquad\qquad\qquad \\ 2x^3 + 4x^2 \qquad\qquad \\ \hline 4x^2 + 7x \qquad \\ -\,4x^2 - 8x \qquad \\ \hline -x + 3 \\ x + 2 \\ \hline 5 \end{array}$$

Thus the quotient is $\boxed{3x^4-x^3-2x^2+4x-1}$

18. A man is walking away from a lamppost with a light source 6 meters above the ground. The man is 2 meters tall. How long is his shadow when he is 110 meters from the lamppost?

Answer: 55 m

By using similarities and letting s be the length of his shadow,

$$\begin{aligned}\frac{2}{6} &= \frac{s}{s+110}\\ \frac{1}{3} &= \frac{s}{s+110}\\ s+110 &= 3s\\ s &= \boxed{55\,m}\end{aligned}$$

19. Find the length of the shorter segment made on side AB of $\triangle ABC$ by the bisector of $\angle C$, if $AB = 20, AC = 12$, and $BC = 18$ cm.
Answer: 8 cm

Let the two parts of AB be n and $20 - n$. Using similarities,

$$\begin{aligned}\frac{20-n}{n} &= \frac{12}{18}\\ \frac{20-n}{n} &= \frac{2}{3}\\ 60-3n &= 2n\\ n &= 12\\ 20-n &= \boxed{8\,\text{cm}}\end{aligned}$$

20. How many different three-digit numbers less than 300 can be formed with the digits 1,2,3 and 5 if repetition of digits is not allowed?
Answer: 12

The numbers are in the form $1**, 2**$. So there are $3P2 + 3P2 = 2(3P2) = \boxed{12}$ numbers that satisfy the condition.

21. Factor completely the expression $x^3 - 7x + 6$.
Answer: $(\mathbf{x-1})(\mathbf{x-2})(\mathbf{x+3})$

- Solution 1
Using the rational root theorem and factor theorem, we find that when $x = 2$,

$$2^3 - 7(2) + 6 = 0$$

So $x - 2$ is a factor. Thus

$$x^3 - 7x + 6 = (x-2)(x^2 + 2x - 3) = \boxed{(x-1)(x-2)(x+3)}$$

- Solution 2

$$\begin{aligned} x^3 - 7x + 6 &= x^3 - x - 6x + 6 \\ &= x(x+1)(x-1) - 6(x-1) \\ &= (x-1)\left(x(x+1) - 6\right) \\ &= (x-1)(x^2 + x - 6) \\ &= \boxed{(x-1)(x-2)(x+3)} \end{aligned}$$

22. An integer between 1 and 10000 inclusive is selected at random. What is the probability that it is a perfect square?
Answer: $\mathbf{\dfrac{49}{5000}}$

From 1 to 10000, there are $\sqrt{10000} = 100$ perfect squares. Since 1 and 10000 are also perfect squares, the numebr of perfect squares between 1 and 10000 is $100 - 1 - 1 = 98$. Thus, the probability of choosing a perfect square is $\dfrac{98}{10000} = \boxed{\dfrac{49}{5000}}$.

23. If a chord 24 cm long is 5 cm from the center of a circle, how long is a chord 10 cm from the center?
Answer: $\mathbf{2\sqrt{69}}$ cm

LEt R be the radius of the circle. Solving for R,

$$\begin{aligned} 5^2 + \left(\frac{24}{2}\right)^2 &= R^2 \\ R &= 13\,\text{cm} \end{aligned}$$

Now, Letting d be the be the length of the chord 10 cm from the center,

$$\begin{aligned} 10^2 + \left(\frac{d}{2}\right)^2 &= R^2 \\ 10^2 + \frac{d^2}{4} &= 13^2 \\ \frac{d^2}{4} &= 69 \\ d &= \boxed{2\sqrt{69}\,\text{cm}} \end{aligned}$$

24. Pipes are being stored in a pile with 25 pipes in the first layer, 24 in the second, and so on. If there are 12 layers, how many pipes does the pile contain?
Answer: 234

Using the sum of arithmetic sequence, $S_{12} = \frac{12}{2}(2(25) + 11(-1)) = \boxed{234}$.

25. Find the solution set of the inequality $x^2 < 5x - 6$.
Answer: $\mathbf{2 < x < 3}$

$$\begin{aligned} x^2 &< 5x - 6 \\ x^2 - 5x + 6 < 0 \\ (x-2)(x-3) < 0 \end{aligned}$$

So we split the cases into 3 intervals: $(-\infty, 2), (2, 3), (3, \infty)$

	$(-\infty, 2)$	$(2, 3)$	$(3, \infty)$
$x - 2$	-	+	+
$x - 3$	-	-	+
$(x-2)(x-3)$	+	-	+

Therefore, $x \in \boxed{(2, 3)}$.

26. Suppose an object is dropped from the roof of a very tall building. After t seconds, its height from the ground is given by $h = -16t^2 + 625$, where h is measured in feet. How long does it take to reach ground level?
Answer: 6.25 s

h should be 0.

$$\begin{aligned} -16t^2 + 625 &= 0 \\ 16t^2 - 625 &= 0 \\ (4t + 25)(4t - 25) &= 0 \\ t &= \boxed{\frac{25}{4}\text{ s}} \end{aligned}$$

27. In $\triangle ABC, AB || DE, AC = 10$ cm, $CD = 7$ cm, and $CE = 9$ cm. Find BC.
Answer: $\mathbf{\frac{90}{7}}$ cm

Since there were no figures shown in the problem, we'll just assume that points D and E are on AC and BC, respectively, for the sake of answering the question.
Using similarities,

$$\begin{aligned}\frac{AC}{CD} &= \frac{BC}{CE}\\ \frac{10}{7} &= \frac{BC}{9}\\ BC &= \boxed{\frac{90}{7}\text{ cm}}\end{aligned}$$

28. Find a polynomial $P(x)$ of degree 3 that has zeroes -2, 0, and 7, and the coefficient of x^2 is 10.
Answer: $\mathbf{P(x) = 10x^3 - 50x^2 - 490x}$

$$P(x) = 10x(x+2)(x-7) = \boxed{10x^3 - 50x^2 - 490x}$$

29. Find the equation (in the form $ax + by = c$) of the perpendicular bisector of the line segment joining the points $(1, 4)$ and $(7, -2)$.
Answer: $\mathbf{x - y = 2}$

Perpendicular bisector - A line that passes through the midpoint of the segment joining to points and perpendicular to the given segment.

$$m = \frac{4 - (-2)}{1 - 7} = -1$$

So the slope of the perpendicular bisector should be 1. Also, the midpoint of the segment is at $(3, 1)$.
Thus, the equation of the perpendicular bisector is

$$y - 1 = x - 3 \to \boxed{x - y = 2}$$

30. The sides of a polygon are 3,4,5,6, and 7cm. Find the perimeter of a similar polygon, if the side corresponding to 5 cm is 9 cm long.
Answer: 45 cm

The perimeter of the given polygon is 25 cm. By similarity,

$$\frac{P}{25} = \frac{9}{5} \to P = \boxed{45\,\text{cm}}$$

31. If $\tan\theta = \dfrac{2}{3}$ and θ is in Quadrant III, find $\cos\theta$.
Answer: $-\dfrac{\mathbf{3\sqrt{13}}}{\mathbf{13}}$

$$\cos\theta = -\frac{3}{\sqrt{2^2 + 3^2}} = -\frac{3\sqrt{13}}{13}$$

32. The first term of a geometric sequence is 1536, and the common ratio is $\frac{1}{2}$. Which term of the sequence is 6?
Answer: 7

$$\begin{aligned} 1536\left(\frac{1}{2}\right)^{n-1} &= 6 \\ (2)^{1-n} &= 2^{-8} \\ 1-n &= -8 \\ n &= \boxed{7} \end{aligned}$$

33. Inf $\triangle ABC$, $\angle B = 2\angle A$. The bisectors of theses angles meet at D, while BD extended meet AC at E. If $\angle CEB = 70°$, find $\angle BAC$.
Answer: 35°

Let $\angle BAC = x$. Now we have $\angle DAE = \frac{x}{2}$, $\angle ABD = x$, so $\angle ADE = \frac{3x}{2}$. Also, $\angle CED$ is a remote exterior angle of $\triangle AED$. Thus,

$$\begin{aligned} \angle DAE + \angle ADE &= \angle CED \\ \frac{x}{2} + \frac{3x}{2} &= 70 \\ x = \angle BAC &= \boxed{35°} \end{aligned}$$

34. Find the sum of the series: $\frac{1}{3} + \frac{2^1}{3^2} + \frac{2^2}{3^3} + \frac{2^3}{3^4} + \cdots$.
Answer: 1

A geometric series where the common ratio r is $\frac{2}{3}$ and first term $a = \frac{1}{3}$.

$$S = \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{2}{3}} = \boxed{1}$$

35. What is the remainder when $x^3 - x + 1$ is divided by $2x - 1$?
Answer: $\mathbf{\frac{5}{8}}$

By the remainder theorem, $2x - 1 = 0 \to x = \frac{1}{2}$. The remainder is

$$\left(\frac{1}{2}\right)^3 - \frac{1}{2} + 1 = \boxed{\frac{5}{8}}$$

36. Triangle ABC has right angle at B. If $\angle C = 30°$ and $BC = 12$ cm, find AB.
Answer: $\mathbf{4\sqrt{3}}$ cm

$$AB = 12\tan 30° = \boxed{4\sqrt{3}\text{ cm}}$$

37. What is the largest root of the equation $2x^3 + x^2 - 13x + 6 = 0$?
Answer: 2

$$\begin{aligned} 2x^3 + x^2 - 13x + 6 &= 0 \\ (x+3)(x-2)(2x-1) &= 0 \\ \max x = \boxed{2} \end{aligned}$$

38. Find the equation (in the form $x^2 + y^2 + cx + dy = e$) of the circle that has the points $(1, 8)$ and $(5, -6)$ as the endpoints of the diameter.
Answer: $\mathbf{x^2 + y^2 - 6x - 2y = 48}$

The midpoint of the diameter, i.e, $(3, 1)$, is the center of the circle and half their distance is the radius, i.e,$\dfrac{\sqrt{(5-1)^2 + (8-(-6))^2}}{2} = \sqrt{58}$.Thus, the equation

$$\begin{aligned} (x-3)^2 + (y-1)^2 &= 58 \\ x^2 + y^2 - 6x - 2y &= 48 \end{aligned}$$

The equation is $\boxed{x^2 + y^2 - 6x - 2y = 48}$

39. The distance from the midpoint of a chord 10 cm long to the midpoint of its minor arc is 3 cm. What is the radius of the circle?
Answer: $\mathbf{\dfrac{17}{3}}$ **cm**

Using the chord-chord theorem,

$$\begin{aligned} (2r-3)(3) &= 5^2 \\ 2r - 3 &= \frac{25}{3} \\ 2r &= \frac{34}{3} \\ r &= \boxed{\frac{17}{3}\text{ cm}} \end{aligned}$$

40. Find the equation (in the form $ax+by=c$) of the line tangent to the circle $x^2+y^2=25$ at the point $(3,-4)$.
Answer: $\mathbf{3x-4y=25}$

The line $ax+by=r^2$ is tangent to the line $x^2+y^2=r^2$ at the point (a,b).
Thus, the tangent line is $\boxed{3x-4y=25}$

41. How many triangles can be formed by 8 points, no three of which are collinear?
Answer: 56

Choosing three points without repetition and without considering the arrangement of the three chosen points, the number of triangles that can be formed is $\binom{8}{3}=\frac{8!}{5!\cdot 3!}=\frac{6\cdot 7\cdot 8}{6}=\boxed{56}$.

42. Two tangents to a circle form an angle of 80°. How many degrees is the larger intercepted arc?
Answer: 260°

Let x be the larger intercepted arc.

$$\begin{aligned}\frac{x-(360-x)}{2} &= 80\\ x-180 &= 80\\ x &= \boxed{260^\circ}\end{aligned}$$

43. Find all real solutions of the equation $x^{\frac{1}{2}}+x^{\frac{1}{6}}-2=0$.
Answer: 1

Let $u=x^{\frac{1}{6}}$

$$\begin{aligned}u^3+u-2 &= 0\\ (u-1)(u^2+u+2) &= 0\\ u &= 1\\ x^{\frac{1}{6}} &= 1\\ x &= \boxed{1}\end{aligned}$$

44. The sides of a triangle are 6, 8, and 10, cm, respectively. What is the length of its shortest altitude?
Answer: $\mathbf{\frac{24}{5}}$ cm

The shortest altitude is the altiude to the hypotenuse.
Which is $h = \dfrac{ab}{c} = \dfrac{6 \cdot 8}{10} = \boxed{\dfrac{24}{5}\text{ cm}}$

45. In a circle of radius 10 cm, arc AB measures $120°$. Find the distance from B to the diameter through A.
Answer: $5\sqrt{3}$ cm

$$d = 10 \sin 60° = \boxed{5\sqrt{3}\text{ cm}}$$

46. Find the remainder when x^{100} is divided by $x^2 - x$.
Answer: x

Let $R(x) = ax + b$ be the remainder. By the remainder theorem, we get:

$$\begin{aligned} R(0) = b &= 0^{100} = 0 \\ R(1) = a + b &= 1^{100} = 1 \end{aligned}$$

Thus, $a = 1, b = 0$. The remainder is $\boxed{x}$.

47. Find the area (in terms of π) of the circle inscribed in the triangle enclosed by the line $3x + 4y = 24$ and the coordinate axes.
Answer: π units^2

Shortcut: Let r be the radius of the inscribed circle, c be the hypotenuse of the right triangle that is formed, and P its perimeter.

$$P = 2(r + c)$$

Since the sides of the right triangle that is formed are of length $3, 4, 5$, $P = 12, c = 5$. Therefore
$r = \dfrac{P}{2} - c = 1$. Which gives an area of $\boxed{\pi\text{ units}^2}$

48. In how many different ways can 5 persons be seated in an automobile having places for 2 in the front seat and 3 in the back if only 2 of them can drive and one of the others insists on riding in the back?
Answer: 12

There are 2 ways to choose a driver, 1 way to choose someone who will seat beside the driver, and $3! = 6$ ways to arrange the passengers at the back. Thus, there are $2 \cdot 1 \cdot 6 = \boxed{12}$ ways.

49. Factor completely the expression $x^4 + 3x^2 + 4$.
Answer: $(\mathbf{x^2+x+2})(\mathbf{x^2-x+2})$

$$\begin{aligned} x^4 + 3x^2 + 4 &= x^4 + 4x^2 + 4 - x^2 \\ &= (x^2+2)^2 - x^2 \\ &= \boxed{(x^2+x+2)(x^2-x+2)} \end{aligned}$$

50. A bag is filled with red and blue balls. Before drawing a ball, there is a $\frac{1}{4}$ chance of drawing a blue ball. After drawing out a ball, there is now a $\frac{1}{5}$ chance of drawing a blue ball. How many red balls are originally in the bag?
Answer: 12

Let r be the number of red balls and b be the number of blue balls. The first condition states that

$$\frac{b}{r+b} = \frac{1}{4}$$

or $r = 3b$. The second condition, on the other hand, can be split into two different cases

- Case 1: The ball that was drawn out is blue

$$\begin{aligned} \frac{b-1}{r+b-1} &= \frac{1}{5} \\ \frac{b-1}{4b-1} &= \frac{1}{5} \\ 5b-5 &= 4b-1 \\ b &= 4 \end{aligned}$$

- Case 2: The ball that was drawn out is red

$$\begin{aligned} \frac{b}{r+b-1} &= \frac{1}{5} \\ \frac{b}{4b-1} &= \frac{1}{5} \\ 5b &= 4b-1 \\ b &= -1 \end{aligned}$$

From the two cases, it shows that the ball that was drawn before the second draw was blue. Which means there are $r = 3(4) = \boxed{12}$ red balls originally.